

# Old firms and the relationship between age and size\*

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If old firms are on average much larger than young firms, does it mean that firms get better with age? Using Danish administrative data on firms aged 0-65, we study the relationship between size and age. In the cross-section, average size is increasing with age. However, exploiting the panel structure through fixed-effects estimation or a partial identification approach to the so-called age-period-cohort problem, we find evidence that firm size increases with age only for the first 10-15 years and falls after that. Moreover, sample composition effects seem to be important to understand the patterns in the cross-section: We find significant differences in exit rates by firm size and strong cohort effects for firms entering in late 1950's. We also find that the exit rate is not monotonically decreasing with age; for the smaller firms it starts increasing again in their 20's, spiking in late 30's.

JEL codes: D22, E23, E24, L11

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# 1 Introduction

There is a growing literature on decline in business dynamism that documents a fall in firm entry ([Hopenhayn et al., 2022](#); [Karahan et al., 2024](#)) and points out that this has implications for employment: young firms generate the majority of new jobs, so these “missing firms” lead to a reduction in aggregate employment ([Davis and Haltiwanger, 2014](#); [Decker et al., 2014](#); [Adelino et al., 2017](#); [Pugsley and Şahin, 2019](#), e.g.). At the same time, the lack of entry pushes the average age of firms in the economy up. The fact that the average firm is getting older can have implications, both for on the micro level for firm behavior as well as in the aggregate. In this paper we present evidence of the firm life cycle and show that firm activity, measured by its size, deteriorates after reaching a certain age, despite the positive correlation between size and age in the cross-section.

Using administrative data where we can track firms of all ages, we show that older firms are on average larger than younger firms and very old firms are on average very large. We find evidence that this is likely driven by differing sample composition across age rather than increasing size as a firm ages: First, larger firms survive at higher rates while smaller firms are more likely to exit. Second, there were particularly strong firms in the cohorts that entered in the late 50’s which are the oldest firms in our sample. After taking these selection issues into account, the average size only rises for the first 10-15 years and falls with age for firms older than that.

We are able to analyze firm life cycle profiles up to a firm age of 65, which is much longer than what is typically done in the literature. The reason is that we exploit a Danish administrative micro data set that provides direct information about firm age. In contrast to other data sets used in the literature, the age information is not truncated and hence very old firms can be studied. For example, one particularly useful source of information about firm dynamics has been the Longitudinal Business Database (LBD).<sup>1</sup> However, in the LBD, 1976 is the first cohort and for all older firms the true age is not recorded, which limits how much one can learn about old firms. In contrast, Danish registry data contains precise information about the starting date and therefore allows to distinguish the effect of aging even among old firms. Furthermore, our data contains the universe of Danish firms of all sizes and all sectors, while other data sets often only contain publicly traded firms which biases the sample towards the largest and the most successful firms.

We first show that in the cross-section, the average firm size increases with firm age.

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<sup>1</sup>See [Jarmin and Miranda \(2002\)](#); [Chow et al. \(2021\)](#).

A 30-year old firm on average is 15% larger than an average 10-year old firm (measured as employment, sales, and value added). However, size increases even more strongly at very old ages: a 60-year old firm is on average almost five times larger than a 30-year old firm. Age could increase firm size by, for instance, building up knowledge about the firm’s competitive environment, human capital or firm-specific skills. Sample composition effects, on the other hand, could increase average firm size if smaller firms are systematically more likely to exit or if older firms in the sample happen to be larger for some other reason than age itself.

Then we use two approaches to decompose the positive relationship between size and age in the cross section: by exploiting the panel structure of the data and by a partial identification approach to the age–period–cohort (APC) problem (Fosse and Winship, 2019). We find that sample composition effects contribute to the positive size-age relationship. Once allowing for firm fixed effects or for the APC structure, the positive relation between average size and age does not persist across the firm life cycle. Instead, average size peaks in the teens (for FE) or even earlier (for APC) and is falling later in the firms life.

When inspecting the estimated firm fixed effects we observe a positive relationship between the average firm fixed effect and age. The estimated firm fixed effects capture any factor that does not change over the firm’s life, such as the unobservable firm quality, as well as many observable factors, such as sectoral affiliation<sup>2</sup> or year of entry, i.e. the firms’ cohort effects. When interpreted as firm quality, it is plausible that firms with higher quality (i.e. larger firms) are less likely to exit. Such non-random exit would then dynamically lead to improvements in quality (i.e. size) in the pool of surviving firms. In order to test for the presence of such effects, we document firm exit rates and how they differ for firms of different sizes at different ages. We report two findings. First, firms in the bottom tercile of size (or quality measured by firm FE) are almost twice as likely to exit compared to those in the first tercile for a given age. Furthermore, for the firms in the bottom tercile the exit rate is not monotonically decreasing with age; after declining strongly until age 10, it starts to increase at age of 20 and peaks at age of 35. This is in contrast to standard stylized facts about firm exit, where the exit rate is assumed to decline uniformly both with size and age (Klette and Kortum, 2004).

Recent literature argued that cohort effect can be particularly important (Hamano and Okubo, 2023; Ma et al., 2025). We use the partial identification approach described by Fosse and Winship (2019) to address the age-period-cohort problem and to speak to cohort

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<sup>2</sup>At least to the extent that most firms do not change their sector of operation.

effects directly. Across all sets of assumptions, one finding emerges: the cohort of firms that entered in the 1950's is a particularly strong one. Given that these are the oldest firms in our sample, these cohort effects contribute to the positive relationship of age and size in the cross-section.

There is a tradition of examining the role of certain firm characteristics such as size and age on its outcomes ([Coad, 2018](#)). In aggregate, changes in the unemployment rate are directly linked to the changes in employment in individual firms and so understanding the characteristics of growing firms is an important question both for academics and for policymakers. This paper contributes to this debate by focusing on the effect of aging. [Evans \(1987\)](#) was among the first to use a comprehensive manufacturing firm data to uncover a negative effect of age on firm employment growth between 1976 and 1980. Relative to this founding study, our data is more granular with respect to age and perhaps due to this fact we are able to uncover a more complex relationship between age and the odds of exiting. [Haltiwanger et al. \(2013\)](#) find that size does not drive employment growth after controlling for age. [Clymo and Rozsypal \(2025\)](#) study the interaction of size and age for cyclicalities of firms. [Dinlersoz et al. \(2024\)](#) examine leverage over the firm life cycle.

Close to the analysis in this paper, [Navaretti et al. \(2014\)](#) use the EFIGE survey combined with the Amadeus database for France, Italy and Spain between 2001 to 2008. Compared to our sample, they study only surviving manufacturing firms with more than 10 workers, whereas we analyze firms from all sectors, both surviving firms and those that exit (which allows us to analyze the likelihood of exit as a function of firm size) and with a median employment equal to 4 full time equivalent workers. They find that firms grow more slowly when they are older. They find that even after including many other variables (such as the age of CEO, number of graduates in workforce, R&D activity, productivity, capital intensity, profitability, finance), the effect of age is still significant. Using a data set of Italian exporting firms, [Grazzi and Moschella \(2017\)](#) find that the positive relationship between export status and growth declines with firm age. In the sample of Irish firms between 1972-2010, [Lawless \(2014\)](#) finds that younger firms grow faster than older firms. Moreover, using the Revenue-enhanced Longitudinal Business Database, [Alon et al. \(2018\)](#) compute age profiles for productivity growth. Compared to results presented in this paper, their highest age group is 11-15, compared to a maximum firm age of 60 in our analysis. The basic pattern of our finding is similar to theirs, but by being able to track firms for much longer we are able to document that the deterioration continues even for very old firms. [Loderer et al. \(2017\)](#) analyze Tobin's  $q$  evolution and find that older firms have

lower growth.

In this paper we also contribute to the recent debate about the role of ex-post shocks versus the role of inherent firm characteristics that are present at firm entry (to the extent that we can proxy for the inherent firm quality with the estimated firm FE). Our results suggest that the larger average size of older firms is driven by inherent firm quality rather than a positive effect of aging. This finding is in line with other recent developments in this area. For example, [Sedláček and Sterk \(2017\)](#) find that firms are heavily affected by the conditions when they start, or [Sterk et al. \(2021\)](#) show that “...even after twenty years, *ex-ante* factors still explain about forty percent of the cohort’s employment dispersion”. We document the changes in the distribution in the firm inherent quality as firm age.

Finally, our APC decomposition generates meaningful heterogeneity in cohort effects, with the firms entering in late 50’s being particularly strong. Here we relate to the recent literature both from the US and from Japan that documented that some cohorts are particularly important when focusing on superstar firms ([Ma et al., 2025](#); [Hamano and Okubo, 2023](#)). The recognition of the importance of the APC problem seems to be a promising avenue for further research. For example, [Adam et al. \(2025\)](#) study the age-profile of mark-ups using this approach.

The paper proceeds in the following steps. Section 2 describes the Danish administrative data set and discusses its advantages over other data sources. Section 3 documents the relationship between size and age in the cross section. Section 4 disentangles the role of aging from selection effects by exploiting the panel structure of the data and reverses the sign of the correlation of size and age for firms older than 15. Here we also examine firm exit and find support for the selection effect: small firms are systematically more likely to exit than larger firms across wide range of ages. Section 5 concludes.

## 2 Data

We draw on firm-level administrative data from Denmark. We use data sets that are collected by Danmarks Statistik (DST), a governmental agency that both collects data itself as well as combines information from other government sources such as information obtained during tax collection. We combine “*Generel firmastatistik*” (FIRM) and “*Regnskabsstatistikken*” (FIRE) registers with additional information about employment from the worker-firm matched data set “*Beskæftigelse for lønmodtagere*” (BFL). We also use FIGT - *Gammel Firmastatistik* and FIGF - *Gammel firmastatistik regnskabsdata* as alternatives

to FIRM and FIRE for the data in the 1990’s. For more information about the data and dataset construction, see Appendix A. Our data set covers the universe of Danish firms between 1992 and 2022. While the raw data contains both active and inactive firms as well as non-employer firms, we apply additional minimal activity and reporting requirements. We define firm exit as the last year when we observe a firm to be active.

**Variables of interest** We focus on two variables that are often used as proxies for firm size: employment and sales. Information about employment is collected via the tax system and it is based on compulsory contributions that every worker (subject to residency registration and minimum annual earnings of roughly 1300 EUR) in Denmark makes to the labor market supplementary pension fund (ATP). Two measures of employment are utilized. For most of the analyses, similar to what is done in the literature, the employment variable (*Antal ansatte (i årsværk)*) captures the firm’s employment evaluated in full-time equivalent units, subject to a minimum activity threshold.<sup>34</sup>

Sales (*Omsætning*) and value added (*Værditilvækst*) come from Accounting statistics (*Regnskabsstatistik*), which is created by DST by combining its own survey and data from SKAT and the Danish Business Authority (DBA, *Erhvervsstyrelsen*), which is an agency under the Ministry of Business Affairs. Both sales and value added are measured in thousands of Danish kroner. Table 1 shows descriptive statistics of our variables of interest. We transform nominal sales to real by dividing it by the aggregate price deflator (in 2021 prices).<sup>5</sup>

**Old firms in the data** Our data set not only contains firms of all sectors and sizes. It also contains direct information on the founding date of each firm. This allows us to analyze the effects of age not only for firms that start within the sample period but for all firms, including the very old. This is in contrast to other data sets that are typically used to investigate firm dynamics. The Longitudinal Business Dataset (LBD), for example, is a confidential data set based on the Business register of the US Census and

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<sup>3</sup>Workers are counted if they worked at least 80 hours in the given year, were not registered as fully unemployed in the last week of November and are residents of Denmark.

<sup>4</sup>For both measures, the primary source of information are the FIRM and FIRE registers. If the information is missing in those, we instead construct and use a measure of headcount from the BFL register.

<sup>5</sup>Dataseries PRIS1121 provided by DST. The value of Danish krone has been fixed to Deutsche Mark from 1982 and to the Euro since its inception in 1999. Since then the exchange rate has been set to kr. 746.038 per 100 euro with fluctuation band of +/- 2.25 per cent. Because of this monetary regime, both interest rates and inflation in Denmark closely track the corresponding values in core European countries.

Table 1: Summary statistics of variables of interest

	N	mean	q25	median	q75	q90	sd	skewness	kurtosis
employment	1.8	15.7	2.0	4.0	9.7	24.0	129.0	76.5	9143.8
sales	1.8	39.8	2.6	5.7	15.5	47.9	613.9	156.0	40254.8
value added	1.7	12.2	1.2	2.5	6.0	16.2	153.7	243.4	136183.6

*Note:* The table reports unconditional descriptive statistics for employment and sales, both in levels and growth rates. Turnover is measured millions of Krone deflated to 2021 values. Employment is measured in number of workers working full time equivalent hours. “N” is the number of firm-year observations in millions, “sd” is the standard deviation.

covers in its current release the period between 1976 and 2016. It contains only employer firms/establishments and does not contain information about the founding date. In that data set, firm age is therefore only known for firms that start within the sample period 1976–2022. This implies that using the LBD, effects of age can only be analyzed up until age 46.<sup>6</sup>

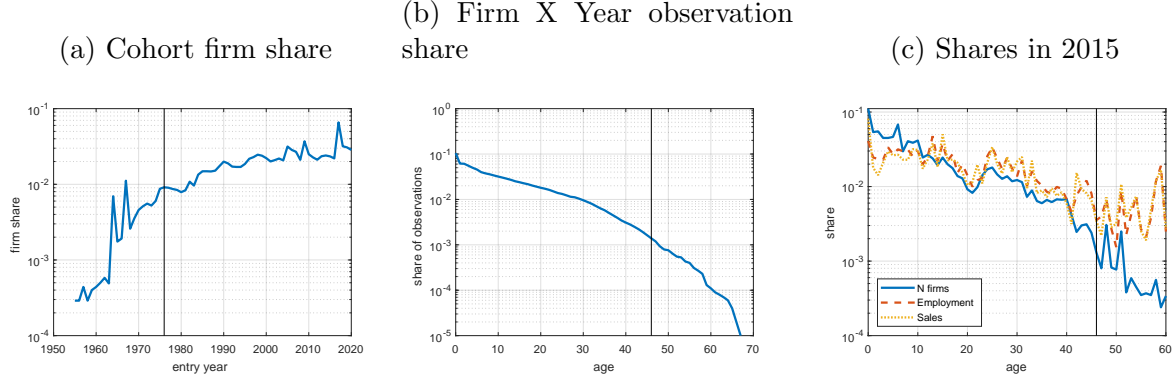
However, firms older than that are a substantial fraction of the population of firms. To illustrate, Figure 1, panel (a), displays the cohort distribution of firms across founding years for all firms across the whole sample. It shows that roughly 10% of all firms that were active in 2015 started before 1976, the earliest observable founding date in the typically used LBD. Moreover, there is a very long right tail of firm age which has been truncated in the graph to facilitate readability: 0.9% of all active firms in 2015 have starting dates prior to 1960, but these firms employed 9.7% workers. Figure 1, panel (b), emphasizes the importance of old firms in the full sample: It displays the cumulative distribution function of all firm-year observations across firm age.<sup>7</sup>

In terms of employment old firms are even more important: In 2015, firms founded before 1976 employed 10% of workers. Across all firm-year observations firms older than age 46 employed 5.4% of all workers. Our data set includes all these firms and is hence

<sup>6</sup>A well used public alternative is the Business Dynamics Statistics (BDS). BDS that is based on LBD and covers the period between 1978 and 2018. However, it only provides aggregated information so individual firms cannot be traced. For example, it is possible to learn how many firms entered in 1980 and how many workers these firms employed, but it is not possible to check how many workers these firms have 10 years later, because the employment of firms aged 6-10 is reported together. In the currently available release (2018), the final age bin is 26-40 (and then all the left-censored firms together).

<sup>7</sup>In the regressions we will restrict the firmst to start after 1955 to make sure that groups defined by combination cohort - sector contain enough observations.

Figure 1: The distribution of entry year and age



*Note:* The figure displays the distribution of firm entry years and age in the sample. Panel (a) plots the cohort share across all firms (unit of observation: firm). Panel (b) plots the age share (unit of observation firm-year). Panel (c) shows the firm, sales and employment share in 2015. All panels' y-axis is in logarithmic scale. Vertical lines show the cutoff where data would have been truncated if the dataset were subject to the same starting year (1976) as in the LBD data set.

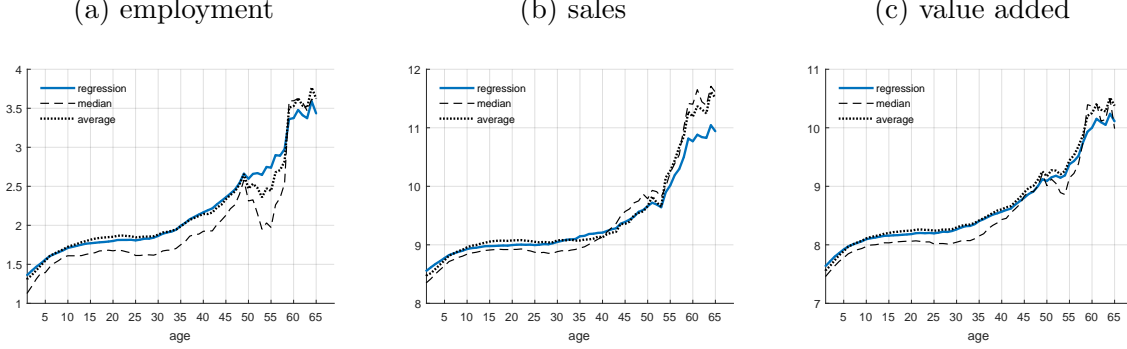
able to capture this important share of economic activity.

### 3 Age-size patterns

In this section we present our empirical findings. First, we set the scene by documenting the general relationship between size and age in the firm cross-section in section 3.1: Firm size on average rises with age. However, this positive correlation does not necessarily imply that individual firms improve as they age. Instead, the observed positive correlation can be driven by differential sample composition at different ages. In section 3.2, we therefore exploit the panel structure of our data and decompose the overall correlation into correlations with age and firm fixed effects. This decomposition reveals that indeed, the sample seems to be composed of increasingly better firms (higher firm size fixed effects) as we move up the firm age distribution. In contrast, the effect of aging for a given firm turns negative on average for all but young firms up to age 10-15. What drives the differential sample composition across the age distribution? We provide evidence for two drivers: Section 3.3 shows that firm exit is not uniform across all firms. Instead, smaller firms are more likely to exit at all ages, providing evidence for non-random sample attrition over time. Section 3.4 further shows that cohort effects seem to play a role, in particular for firm cohorts that started in the late 1950s. We conclude the empirical evidence with a



Figure 2: Cross-section age profiles



*Note:* This figure shows the unconditional mean and median age pattern (black) as well as the predicted age pattern based on regression (1) (blue). Variables in log-levels.

discussion in section 3.5.

### 3.1 Age-size pattern in the cross-section

What do old firms look like? In this section we document how the average size of firms changes with age. First, we plot the average and the median of the unconditional size distribution (size measured by employment or sales). To eliminate the effect of short term fluctuations and sectoral differences (both in terms of levels but also in the composition of the pool of all firms), we also show the predicted size from the following regression:

$$Y_{it} = \mu + \sum_{a=0}^{N_a} \alpha_a 1_{(A_{it}=a)} + \sum_{p=0}^{N_p} \pi_p 1_{(t=p)} + \sum_{s=0}^{N_s} \sigma_s 1_{(S_{it}=s)} + \delta D_{it} + \varepsilon_{it}, \quad (1)$$

where  $Y_{ist}$  is the variable of interest for firm  $i$  in time period  $t$  and  $\alpha_a$ ,  $\pi_p$ , and  $\sigma_s$  are the  $a$ -th age,  $p$ -th period and  $s$ -th sector fixed effect. Finally,  $D_{it}$  is an indicator function capturing firm exit to capture the fact that firms mechanically shrink in the year in which they exit. As we present in Section 3.3, exit rates are correlated with firm age, so we include  $D$  to avoid the changes in the exit rate (as firms grow older) to be picked up by the age pattern  $\alpha_a$ . Using the estimated coefficients, we predict  $Y$  across all ages for an average value of  $s$  and  $p$  in the sample. The results are depicted in Figure 2. Tables with regression coefficients are listed in Appendix B.

Figure 2 shows that older firms are on average much larger. This is particularly true for the very old firms: While the average firm size is increasing across the whole age distri-

bution, all measures of firm size (employment, sales, and value added) increase especially strongly for firms older than 50 years.

Does the positive correlation between size and age in the cross-section mean that *individual* firms get better as they get older? It is important to realize that this pattern does not imply that *the effect* of aging is positive. Average firm size can increase with age for at least two reasons: On the one hand, it is possible that the firms become bigger as they age. This could be, for instance, because firms gain experience in their market, their employees develop firm-specific skills and increase their productivity over time (Caplin et al., 2022), or they establish and grow their supplier and customer base, or they keep growing out of their financial constraints (Ottonello and Winberry, 2024) etc. On the other hand, the average firm size can also rise with age if the sample composition changes with age in some systematic way. In the next section, we therefore add firm fixed effects to disentangle such unobserved heterogeneity from the age pattern.

### 3.2 Firm FE estimation

In the previous section we documented that the average firm size increases with age. In this section we now disentangle whether this is because firms become bigger as they age or whether this is influenced by differential sample composition across the age distribution. To do that we estimate the following fixed effects model:

$$Y_{it} = \mu_i + \sum_{a=0}^{N_a} \alpha_a 1_{(A_{it}=a)} + \phi x_t + \delta D_{it} + \varepsilon_{it}, \quad (2)$$

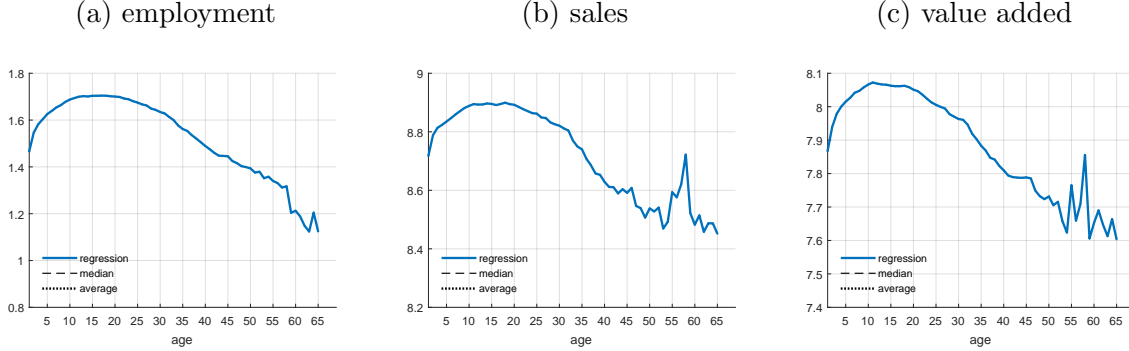
This is a version of regression equation 1, where the intercept  $\mu$  is replaced with firm fixed effect  $\mu_i$ , which also absorbs the sectoral effects  $\sigma_s$ . To avoid the classical *age-period-cohort* identification problem, which will be addressed directly in section 3.4, we do not control for time variation using time fixed effects but instead we include the aggregate GDP growth rate,  $x_t$ .<sup>8</sup>

Once we allow for differing inherent firm quality in the form of firm fixed effects, the pattern of size across age changes. As Figure 3 shows, average firm size is increasing during the first 10 years, mostly flat for the next 10 years and trending down slowly afterwards.

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<sup>8</sup>An alternative to the fixed effects model would have been a random effects model. However, the random effects model is only unbiased under the assumption that the random effects (firm quality) is uncorrelated with the regressors (age). Under the assumption that firm exit is non-random this condition would be violated, so that a random effects model cannot be justified.

Figure 3: Age profiles with firm fixed effects



*Note:* The figure displays the predicted age pattern from regression (2). Shaded areas (very small) refer to 95% confidence intervals. Sales and value added are measured in thousands of (2021) Danish kroner. Variables in log-levels.

This inverse U shape is very different from the unconditional specification showed in Figure 2, where we showed that older firms are on average larger.

Next, we investigate the firm quality, measured by the firm fixed effects. Our dataset is an unbalanced panel. The implication of this is that, for example, if we look at the two sets of firms that are characterized by two different values for firm age (for example set A contains all firms that in year 2015 were 10 years old, and set B contains firms that were 11 years old), these sets will not be identical. This allows us to ask, how *the average* firm quality changes across age, which we measure by the difference between *the average* firm fixed effects.

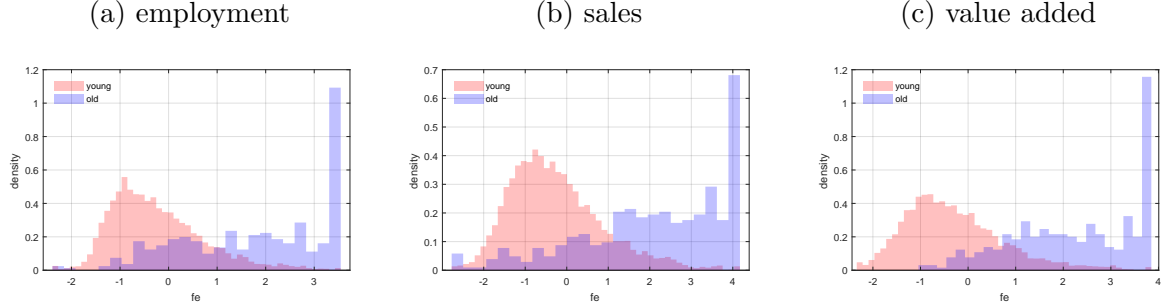
Figure 4 gives a first impression of the changes in sample composition—it shows histograms of the fixed effects estimated from Equation (2) for two distinct age groups of firms active in 2015: young firms (3–5 years) and old firms ( $\geq 50$  years). We find that the distribution of firm quality shifts to the right for older firms.<sup>9</sup> An alternative way of visualizing the differences in the pool of firms is to track the average firm quality by age. Figure 5 displays the average firm fixed effect estimated from Equation (2) for all firms that are active at a particular age.<sup>10</sup> We find that the average firm quality increases with age. This is particularly pronounced for the very old firms: The age-gradient becomes much steeper around age 50.

To summarize, the unconditionally positive correlation between size and age in the

<sup>9</sup>This finding is robust to different age cutoffs and years, see Appendix C.3.

<sup>10</sup>To interpret this figure, it is important to remember that firms are in the sample for multiple years so that each firm contributes to the averages of all ages at which it is active.

Figure 4: Firm quality histograms



*Note:* The figure displays histograms of the firm fixed effects  $\hat{\mu}_i$  estimated from equation (2) of firms active in 2015 and part of two age groups: the ‘young’ firms of age 3–5 years (red) and the ‘old’ firms of 50 years and older (blue). Outcome variables are employment (panel (a)), sales (b) and value added (c). Firm fixed effects are winsorized at 0.5 and 99.5%.

cross-section can be decomposed into a negative relationship between size and age after age 10-15 and a positive relation between the average firm fixed effect and age. What drives the latter effect? In the next two sections we examine two mechanism that can contribute this differential sample composition across the age distribution.

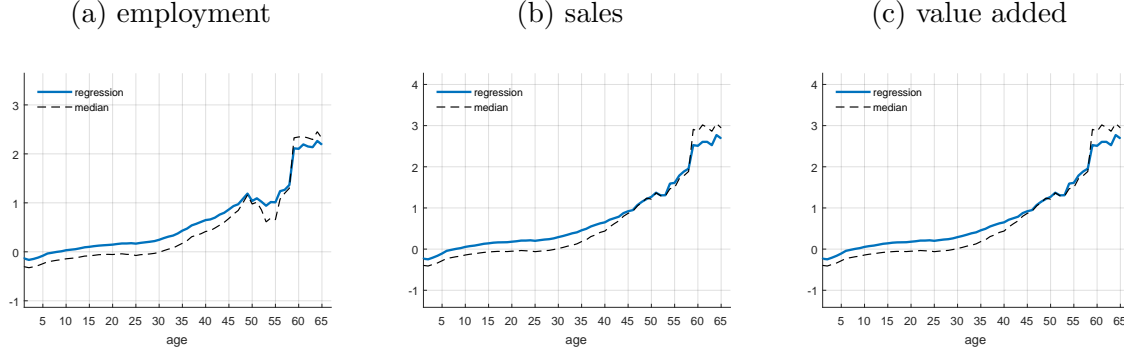
### 3.3 Exit rate heterogeneity over size and age

One reason for a differential sample composition across the age distribution can be non-random firm exit. If not all firms are equally likely to exit, then dynamically, the sample of surviving firms at a given age will differ from the firms active at lower ages. In this subsection we present evidence that firm exit is indeed not uniform across all sizes but that it is more likely for smaller firms at any given age to exit. We observe this pattern when we use either the current firm size or firm quality captured by its firm fixed effect as measure of size. Specifically, we show that “better” firms are less likely to exit. Selection then drives average size up.

To analyze whether exit rates are systematically different for firms in different parts of the firm size distribution, we estimate the exit probability of a firm using the following linear probability model:

$$D_{it} = \mu + \sum_{j=1}^3 \sum_{a=0}^{N_a} \alpha_a 1_{(A_{it}=a)} \times Q_{it}^Y(j) + \sum_{s=0}^{N_s} \sigma_s 1_{(S_{it}=s)} + \sum_{p=0}^{N_p} \pi_p 1_{(t=p)} + \varepsilon_{it}, \quad (3)$$

Figure 5: Age profiles of firm fixed effects



*Note:* The figure displays the average and the median fixed effects estimated from regression (2) where the outcome variables are employment (panel (a)), sales (b) and value added (c) in log levels.

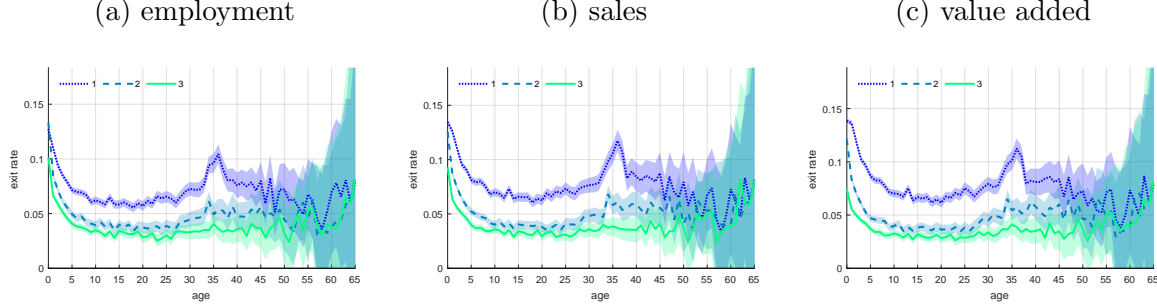
where  $Q_{it}^Y(j)$  is the indicator function of firm  $i$  belonging to  $j$ -tercile of variable  $Y$  in year  $t$ . As before we control for sector and time fixed effects. In Appendix C.4 we show that the results are similar if we use terciles of the firm fixed effect distribution  $\hat{\mu}_i$  instead of current size. To construct the groups in the size distribution we categorize each firm  $\times$  year observation into a corresponding third of the size distribution based on the variable of interest (employment, sales, or value added). The terciles groups are constructed within sector-age groups. The model thus estimates the odds that a firm from a particular segment of the distribution exits at a given age, conditional on surviving until that age. Figure 6 shows the estimated exit probabilities.<sup>11</sup>

The exit probability estimates are tight enough to separate the exit probabilities of firms in different size groups until firms reach their mid 40's. After that, the confidence intervals get too wide to draw any conclusion. We find that the probability of firm exit is indeed systematically different for firms in different parts of the size distribution. *Until firms reach the age of 20*, the patterns are consistent with the stylized facts from the literature Klette and Kortum (2004); smaller firms are more likely to exit than larger firms and older firms are less likely to exit than younger firms.

However, the pattern changes for firms older than 20 years and more so the smaller firms are: exit rates no longer decrease. In fact, they start to increase, especially in their

<sup>11</sup>The differences in the overall levels of exit rates across variables is due to non-reporting firms. As reported in table 1, there are differences in the number of firms that are reporting positive values for the different size variables. Given that reporting is done ex post, one possible reason for not reporting is that a firm is in liquidation: Firms in their final year are less likely to report data. For this reason, we extend the definition of exit and we focus on firms exiting in the current year or in the next two years.

Figure 6: Exit rates by position in the firm distribution



*Note:* The figure displays the predicted exit probabilities by age from equation (3) for different size terciles (first tercile: dotted blue line, second tercile: dashed dark green line, third tercile: solid light green) for employment (panel (a)), sales (b) and value added (c). The shaded areas correspond to 95% confidence intervals.

early thirties, reaching the peak in late 30's. From that age the exit rates start to fall.

Importantly, the pattern with respect to size does not reverse: smaller firms are more likely to exit across all ages.<sup>12</sup> In terms of magnitude, a firm in the smallest third of the size distribution is 1.5-4 times more likely to exit than a firm in the largest third (depending on a particular age bin). This supports the conjecture that the increase in the average firm size is at least partially driven by smaller firms being more likely to exit, changing the composition towards larger firms.

### 3.4 Cohort effects

In the previous subsection we presented evidence for non-random firm exit, implying that smaller firms are systematically more likely to exit than larger firms. Through this dynamic selection, the sample is therefore likely to consist of larger and larger firms on average as we move along the age distribution. In this subsection, we now provide evidence for a second channel that can affect the sample composition: cohort effects. As a firm's cohort, defined as the starting year of the firm, does not change over time, cohort effects are part of the firm fixed effects that we estimated in equation (2). This is particularly striking at the right tail of the age distribution: As firms that belong to recent cohorts are not yet observed at older ages, the firm fixed effects for older ages are estimated based on firms from fewer cohorts. If some of these cohorts turn out to be characterized by particularly large firms, this would show up as higher firm fixed effects at corresponding old ages.

<sup>12</sup>Apart from the first couple of years if we use the firm FE instead of size, see Figure 13.

The problem of jointly estimating the age, period and cohort effects has long been appreciated and across different disciplines (O’Brien, 2015). As Fosse and Winship (2019) illustrate in their overview article, the identification problem of age-period-cohort is limited to linear trends and does not affect non-linear components. One can split the age, cohort and period effects into linear trend and non-linear deviations from these three trends. The crucial insight is that it is really only the linear trend components that give rise to the identification issues. We provide an overview of the methodology in Appendix C.5. We operationalize this approach by estimating the following Equation:

$$Y_{it} = \mu + \omega_1 A_i + \sum_{a=0}^{N_a} \alpha_a 1_{(A_{it}=a)} + \omega_2 t + \sum_{p=0}^{N_p} \pi_p 1_{(t=p)} + \sum_{c=0}^{N_c} \gamma_c 1_{(C_i=c)} + \sum_{s=0}^{N_s} \sigma_s 1_{(S_{it}=s)} + \delta D_{it} + \varepsilon_{it}, \quad (4)$$

where  $\omega_1$  and  $\omega_2$  estimate two linear trends that are the combination of three underlying trends (age, time and cohort) and  $\alpha_a$ ,  $\pi_p$  and  $\gamma_c$  capture the nonlinear components of the age, period and cohort patterns. To ensure that the three linear trends are only absorbed by  $\omega_1, \omega_2$ , we make the following additional constraints:

- two constraints on  $\alpha_a$ ’s:
  - $\sum_{a=0}^{N_a} \alpha_a = 0$ , so that  $\alpha_a$ ’s only capture the deviation from the linear trend estimated by  $\omega_1$
  - $\sum_{a=0}^{N_{a_1}} \alpha_a = \sum_{a=N_{a_1}+1}^{N_a} \alpha_a$ , so there is no remaining trend  $\alpha_a$ ’s
- two constraints of  $\pi_p$ ’s
  - $\sum_{p=0}^{N_p} \pi_p = 0$ , so that  $\pi_p$ ’s only capture the deviation from the linear trend estimated by  $\omega_2$
  - $\sum_{p=0}^{N_{p_1}} \pi_p = \sum_{p=N_{p_1}+1}^{N_p} \pi_p$ , so there is no remaining trend  $\pi_p$ ’s

We choose  $N_{a_1}$  and  $N_{p_1}$  such that the ages and periods are split in half, resulting in all the trend being loaded into  $\omega_1$  and  $\omega_2$  and none being absorbed in  $\alpha_a$ ’s and  $\pi_p$ ’s.

With these constraints, Equation (4) can be estimated and all parameters are fully identified. In order to get from  $\omega_1$  and  $\omega_2$  to  $\alpha$ ,  $\pi$  and  $\gamma$  (the true underlying age, period

and cohort linear trends, respectively), we have to impose additional assumptions.<sup>13</sup> We explore four cases:

1. no linear time trend, forcing  $\pi = 0$ , implying  $\alpha = \omega_1 + \omega_2$  and  $\gamma = \omega_2$
2. no linear trend in cohorts, forcing  $\gamma = 0$ , implying  $\alpha = \omega_1$  and  $\pi = \omega_2$
3. no linear trend in age, forcing  $\alpha = 0$ , implying  $\gamma = -\omega_1$  and  $\pi = \omega_1 + \omega_2$
4. finally, assuming that there is no combined effect of aging *after age of A*, which we implement by forcing  $\alpha = -\bar{\alpha}$  where  $\bar{\alpha}$  is equal to the average non-linear term  $\tilde{\alpha}$  above age  $A$ , implying  $\gamma = -\bar{\alpha} - \omega_1$  and  $\pi = \omega_2 + \omega_1 + \bar{\alpha}$

Here, we present only the full results with our preferred specification (case 1). We prefer this specification as delivers a pattern for time effects that mimics the evolution of the aggregate business cycle with a slight upward trend for both employment and sales. This positive time trend makes intuitive sense as the positive sales can be explained by productivity growth and the positive employment trend is consistent with large labor force increase driven by retirement reforms and net inflow of workers from the abroad. We find the alternative assumptions less convincing since they lead to implausible patterns for the period effects. We report those estimates, together with results for the non-linear components only, in Appendix C.5.

Figure 7 shows the results. Two things are particularly worth noting. First, even with this alternative approach to the fixed-effects estimation, the estimated age pattern remains very similar: on average, firm size increases only for the first 10 years of a firm's life-cycle and decreases afterwards. Second, there are strong cohort effects: Firms that entered during the 1950s are substantially larger than firms that entered afterwards. Since these are exactly the firms that we can observe at very old ages, these cohort effects seem to contribute to the particularly large average firm fixed effects that we documented in Section 3.2.

### 3.5 Discussion

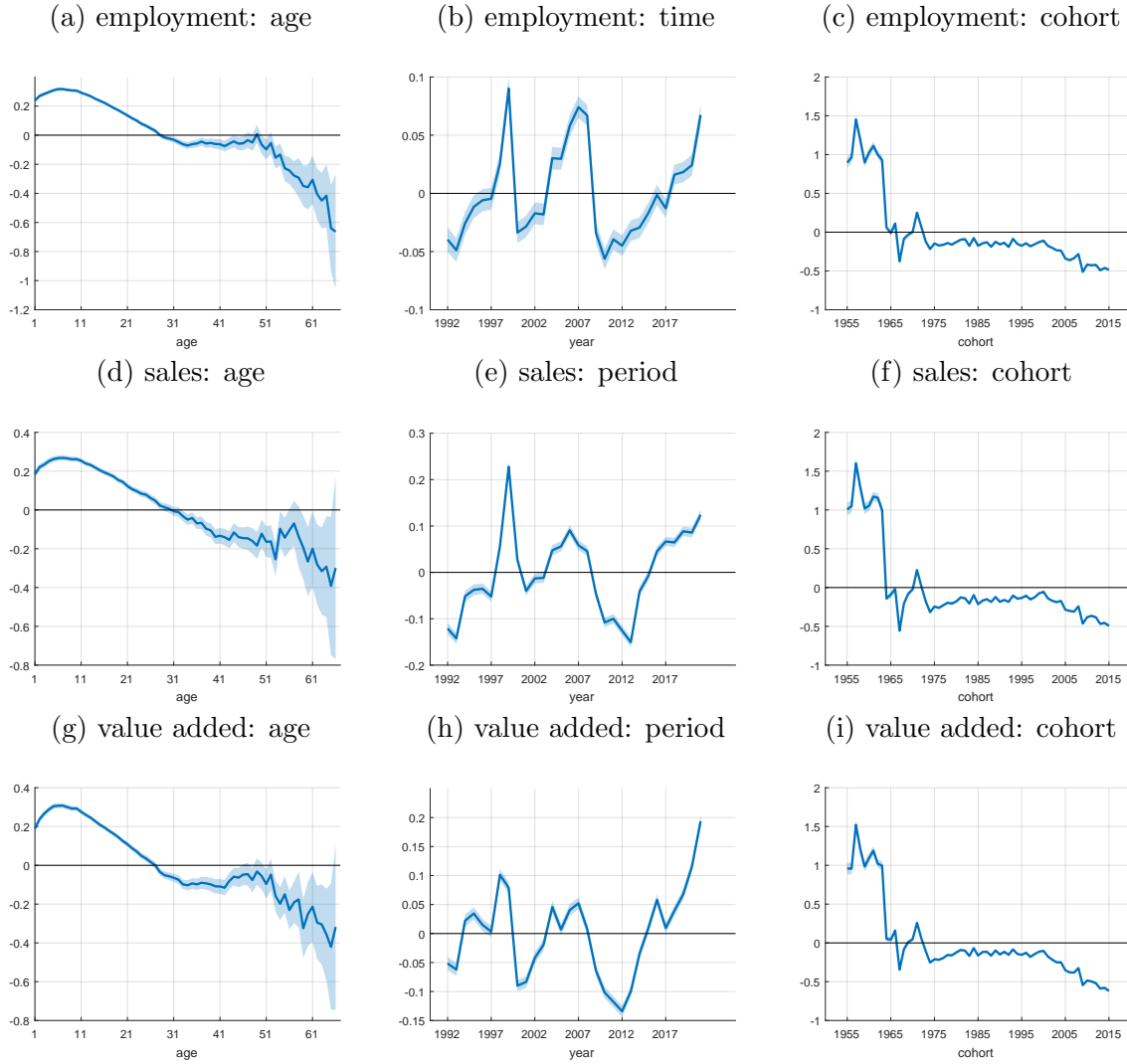
In this paper, we first showed that in the cross-section, the relationship between firm size and age is positive across all ages. However, once we imposed more structure, either by including firm fixed effects or by addressing the age-period-cohort estimation problem, the

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<sup>13</sup>Fosse and Winship (2019) show that  $\alpha - \gamma = \omega_1$  and  $\gamma + \pi = \omega_2$



Figure 7: APC estimation results: assuming no linear trend in period



*Note:* This Figure show the results of estimating Equation (4), under the additional assumption scheme 2 (no linear time trend). The shaded areas correspond to 95% confidence intervals capturing the uncertainty about the non-linear components.

relation between size and age is no longer positive across all ages. Instead, the average size peaks between ages 10-15 after which the correlation turns from positive to weakly negative. We argued that the discrepancies in documented age patterns can be attributed to differential sample composition across the age distribution.

We have found evidence in favor of two mechanisms that likely contribute to this differential sample composition across age: first, dynamic selection driven by non-random exit whereby the smallest firms are more likely to exit, and second, cohort effects, where the oldest firms belong to cohorts with particularly large firms.

So far, we interpreted the firm fixed effects  $\mu_i$ , as estimated by regression equation (2), as a measure of firm quality. Econometrically, it captures the influence of any factor that is constant across all observations the firm is present in the dataset. Some of these characteristics are unobservable such as firm culture or quality of the management within the firm (to the degree these do not change). Some other characteristics are observable, such as firm cohort effects or sectoral differences. In the firm fixed effects specification, all these are lumped together, but the results of the APC estimation suggest that cohort heterogeneity plays an important role.

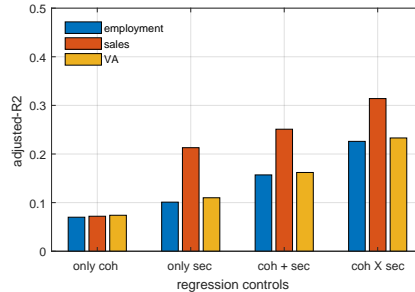
To try to understand the relative contribution of the unobserved quality relative to the observable differences such as sectoral or cohort effects, we run the following regression:

$$\hat{\mu}_i = const + \sum_{s=0}^{N_s} \sigma_s 1_{(S_{it}=s)} + \sum_{c=0}^{N_c} \gamma_c 1_{(C_i=c)} + \sum_{c=0}^{N_c} \sum_{s=0}^{N_s} \rho_{s,c} 1_{(S_{it}=s, C_i=c)} + \varepsilon_i. \quad (5)$$

On the left hand side we use the estimated firm fixed effect from Equation (2), obtaining the results for employment, sales, and value added separately. Equation (5) summarizes four different specifications; cohort effects only, sectoral effects only, sectoral and cohort effects entering additively and finally, having different cohort effects for every sector (sectoral and cohort effects entering multiplicatively). We plot the resulting adjusted  $R^2$  in Figure 8.

The results suggest an important role for unobservable firm quality. Cohort information alone can explain between 5-10% of the variation in firm FE. Sectoral controls alone can explain 10-20% (more for sales than for employment or value added). Using the additive specification explains roughly 15-25%, basically the sum of the previous two cases. The final multiplicative specification is able to explain between 20-30% of the variation. This suggests that there is some heterogeneity in the cohort effects across sectors. This is in line with the findings in [Ma et al. \(2025\)](#) who document a difference between services vs manufacturing in terms of cohorts which gave rise to superstar firms.

Figure 8: Firm fixed effect  $R^2$  decomposition



*Note:* This figure shows the increase in adjusted- $R^2$  from 4 different versions of regression 5, where we vary the regressions on the right hand side: we use either (a) cohort controls only, (b) sector controls only, (c) cohorts and sector controls entering additively or (d) cohort and sector controls entering multiplicatively.

## 4 Conclusion

There is growing evidence on the decline of business dynamism in the US (Hopenhayn et al., 2022; Karahan et al., 2024) and across many developed economies (Calvino et al., 2020) and also in the Latin America (Ayres and Marcos, 2024). If these trends continue, we might find ourselves in an economy, where firms are on average much older than in the past. Should we expect the behavior of the aggregate economy to change as firms become older?

A path to answering this question starts with a better understanding of the differences between old and young firms. The first contribution of this paper is to establish the age patterns for firms much older than possible using other datasets, such as LBD. Second, we show that while the general correlation between age and size is unconditionally positive in the cross-section, once more structure is imposed, the average size starts to decline in their teens at the latest, depending on the methodology and the variable of interest. We document two reasons why this might be happening. First, higher exit of smaller firms changes the pool of firms towards larger firms. Second, the cohorts of firms that started in late 1950's happened to be particularly large. Since firms that started in the 1950's are very old by now, these cohort effects contribute to older firms being larger on average.

We also document a violation of one of the stylized facts in the firm dynamics literature. It is generally believed that older firms exit less than young firms and that smaller firms exit more than larger firms (Klette and Kortum, 2004). Using our data we find evidence that suggests that for the small firms, the exit rate pattern is actually not uniformly decreasing

with age. We show that while it indeed declines strongly in the first ten years, it stabilizes in the following ten years only to start increasing again from age of twenty, experiencing very strong increase after age 30 to peak at 35-37 and start falling afterwards. We do not have a direct explanation for this phenomena, but our speculation would be it has to do with the founder of the firm retiring and these small firms not being economically viable enough to survive the retirement of their original founder.

One challenge is that, short of having a time machine, it is not possible to establish a causal relationship of aging on firm characteristics using some sort of natural experiment. However, we would argue that one can learn a lot from the age patterns, provided one is being cognizant of the underlying identification challenges.

Going forward, there are two avenues that we believe are worth visiting. First, it would be interesting to expand our horizons to even older firms. We stayed limited to firms that started in in the late 50's. This is because the number of firms that survive all the way to 1992 where our dataset starts is naturally getting too small, especially when grouped by all the control variables. However, we believe that there is a way to adapt the econometric framework by making the cohort definition more flexible, allowing for pooling of firms that started in several years long time in the past while keeping the definition of cohort as a single year for more recent entrants. Second, given the large heterogeneity in firm outcomes, it might be insightful to move beyond analyzing averages to get a more comprehensible understanding of the whole distribution ([Coad and Rao, 2008](#); [Coad et al., 2016](#)).

There are policy implications. If the population of established firms is on average better than the population of entrants, then it might be worth to target support to established firms against the young. However, for firms of the same size, it might be better to save a young firm rather than an old firm because the young one is likely to get larger over time. Indeed, in the response to the COVID-19 recession, policy makers around the world considered various policies to support firms that have different short and long run benefits. The trade-off between saving young and small but growing firms and larger but older firms is not obvious and to fully appreciate it one has to take the growth patterns of firms into account.

Another implication of our findings concerns the quantitative theory literature. Workhorse macro models of firms typically abstract from life cycle patterns. In richer firm dynamics models, such as [Bilbiie et al. \(2012\)](#); [Clementi and Palazzo \(2016\)](#), endogenous exit occurs when an exogenous persistent process driving profits falls so much that future

discounted profits are negative. Firm age, however, does not matter except for some financial friction that young firms eventually grow out of. Modeling the firm life cycle is more common in finance. [Mueller \(1972\)](#) proposed a theory that firms follow a S-shaped growth pattern of slow growth at the beginning, high growth at maturity and then an eventual slowdown due to losing their competitive advantage. This pattern has implications for dividend choices that can be tested and are supported by empirical evidence ([Fama and French, 2001](#); [DeAngelo et al., 2006](#)). In the light of our findings it seems crucial to model the micro foundation of both non-random exit and the negative effect of aging past maturity.

## References

- Adam, K., Renkin, T., and Zullig, G. (2025). Markups and marginal costs over the firm life: Implications for the optimal inflation target. Technical report, Discussion Paper Series – CRC TR 224.
- Adelino, M., Ma, S., and Robinson, D. (2017). Firm age, investment opportunities, and job creation. *The Journal of Finance*, 72(3):999–1038.
- Alon, T., Berger, D., Dent, R., and Pugsley, B. (2018). Older and slower: The startup deficit’s lasting effects on aggregate productivity growth. *Journal of Monetary Economics*, 93:68–85. Carnegie-Rochester-NYU Conference on Public Policy held at the Stern School of Business at New York University.
- Ayres, J. and Marcos, M. A. T. (2024). Business dynamism is on the decline globally. online blogpost, IBD, <https://blogs.iadb.org/ideas-matter/en/business-dynamism-is-on-the-decline-globally>.
- Bilbiie, F. O., Ghironi, F., and Melitz, M. J. (2012). Endogenous entry, product variety, and business cycles. *Journal of Political Economy*, 120(2):304–345.
- Calvino, F., Criscuolo, C., and Verlhac, R. (2020). Declining business dynamism: structural and policydeterminants. Technical Report 94, OECD Directorate for Science, Technology and Innovation.
- Caplin, A., Lee, M., Leth-Petersen, S., Saeverud, J., and Shapiro, M. D. (2022). How

- worker productivity and wages grow with tenure and experience: The firm perspective. Working Paper 30342, National Bureau of Economic Research.
- Chow, M. C., Fort, T., Goetz, C., Goldschlag, N., Perlman, B., Stinson, M. H., and White, K. (2021). Redesigning the longitudinal business database. WP 28839, NBER.
- Clementi, G. L. and Palazzo, B. (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics*, 8(3):1–41.
- Clymo, A. and Rozsypal, F. (2025). Firm cyclicalities and financial frictions. techreport.
- Coad, A. (2018). Firm age: a survey. *Journal of Evolutionary Economics*, 28:13–43.
- Coad, A. and Rao, R. (2008). Innovation and firm growth in high-tech sectors: A quantile regression approach. *Research Policy*, 37(4):633–648.
- Coad, A., Segarra, A., and Teruel, M. (2016). Innovation and firm growth: Does firm age play a role? *Research Policy*, 45(2):387–400.
- Davis, S. J. and Haltiwanger, J. (2014). Labor market fluidity and economic performance. Working Paper 20479, National Bureau of Economic Research.
- DeAngelo, H., DeAngelo, L., and Stulz, R. M. (2006). Dividend policy and the earned/contributed capital mix: a test of the life-cycle theory. *Journal of Financial Economics*, 81(2):227–254.
- Decker, R., Haltiwanger, J., Jarmin, R., and Miranda, J. (2014). The role of entrepreneurship in us job creation and economic dynamism. *Journal of Economic Perspectives*, 28(3):3–24.
- Dinlersoz, E., Kalemli-Ozcan, S., Hyatt, H., and Penciakova, V. (2024). Leverage over the Life Cycle and Implications for Firm Growth and Shock Responsiveness. NBER Working Papers 25226, National Bureau of Economic Research, Inc.
- Evans, D. S. (1987). The relationship between firm growth, size, and age: Estimates for 100 manufacturing industries. *The Journal of Industrial Economics*, 35(4):567–581.
- Fama, E. F. and French, K. R. (2001). Disappearing dividends: changing firm characteristics or lower propensity to pay? *Journal of Financial Economics*, 60(1):3–43.

- Fosse, E. and Winship, C. (2019). Bounding analyses of age-period-cohort effects. *Demography*, 56:1975–2004.
- Grazzi, M. and Moschella, D. (2017). Small, young, and exporters: New evidence on the determinants of firm growth. *Journal of Evolutionary Economics*, 28:125–152.
- Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2013). Who Creates Jobs? Small versus Large versus Young. *The Review of Economics and Statistics*, 95(2):347–361.
- Hamano, M. and Okubo, T. (2023). The macroeconomic dynamics of generations of firms. Working Papers 2307, Waseda University, Faculty of Political Science and Economics.
- Hopenhayn, H., Neira, J., and Singhanian, R. (2022). From population growth to firm demographics: Implications for concentration, entrepreneurship and the labor share. *Econometrica*, 90(4):pp. 1879–1914.
- Jarmin, R. S. and Miranda, J. (2002). The longitudinal business database. Technical report, Census Bureau.
- Karahan, F., Pugsley, B., and Şahin, A. (2024). Demographic origins of the start-up deficit. *American Economic Review*, 114(7):1986–2023.
- Klette, T. J. and Kortum, S. (2004). Innovating firms and aggregate innovation. *Journal of Political Economy*, 112(5):986–1018.
- Lawless, M. (2014). Age or size? contributions to job creation. *Small Business Economics*, 42:815–830.
- Loderer, C., Stulz, R., and Waelchli, U. (2017). Firm rigidities and the decline in growth opportunities. *Management Science*, 63(9):3000–3020.
- Ma, Y., Pugsley, B., Qin, H., and Zimmermann, K. (2025). Superstar firms through the generations. Technical report, wp.
- Mueller, D. C. (1972). A life cycle theory of the firm. *The Journal of Industrial Economics*, 20(3):199–219.
- Navaretti, G. B., Castellani, D., and Pieri, F. (2014). Age and firm growth: evidence from three european countries. *Small Business Economics*, 43(4):823–837.

- O'Brien, R. (2015). *Age-Period-Cohort Models Approaches and Analyses with Aggregate Data*. Routledge.
- Ottonello, P. and Winberry, T. (2024). Capital, ideas, and the costs of financial frictions.
- Pugsley, B. W. and Şahin, A. (2019). Grown-up Business Cycles. *The Review of Financial Studies*, 32(3):1102–1147.
- Sedláček, P. and Sterk, V. (2017). The growth potential of startups over the business cycle. *American Economic Review*, 107(10):3182–3210.
- Sterk, V., Sedlacek, P., and Pugsley, B. (2021). The nature of firm growth. *American Economic Review*, 111(2):547–79.



## A Data appendix

**General information about the data** We use a three digit sectoral classification.<sup>14</sup> We drop firms from sectors with lots of publicly administered companies (public administration, defense, education, health care, theaters, concert halls, libraries), as well as firms that are listed as “extra-territorial bodies” and “households as employers”. For 2015, after cleaning and eliminating public sector entities, we have information about 63 thousand firms employing and 1.03M million workers (in FTE units).

**Data cleaning description** In general, a firm is considered active by DST if it engages in a minimum level of economic activity, yet it includes a large number of non-employer firms that can potentially be entities set-up by other firms used for various tax optimization or other administrative reasons. Also, it included many public sector entities such as various entities set up by the national or local governments that are not subject to economic incentives and as such are not the subject of this paper.

To minimize the influence of such entities, we clean our dataset in the following way. First, we remove all entities that are not commercial firms (based on variable `JUR_VIRK_FORM`). Second, we eliminate all firms below a minimal activity threshold. To be considered active, any firm needs to satisfy two criteria: 1) it has to *at some point in its life* have sales over 500k DKK, VA over 200k DKK, and employed more than 1 worker (not necessarily in the same year). 2) it also has to, in any active year, have higher VA than 100k DKK, sales higher than 250k DKK, more than 1 FTE worker. For perspective, 100 DKKr is equivalent to 13.4 EUR or 15 USD.

Firms in the sample that have missing information about the variables of interest could be inactive (in which case ignoring them would not be a problem), or active but not reporting. Given that smaller, less established firms are potentially more likely to not report, ignoring them would bias the results.

The raw dataset includes all entities registered in the firm registry CVR. This includes non-employer firms and many entities created and run by the government (from the municipal level all the way up to the national level). These different entities might have very different optimization problems and hence also behave very differently to standard commercial firms. Also, some entities might be set up for various tax optimization schemes.

Due to changes in the coverage of different sectoral classifications over the 30 years of

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<sup>14</sup>The details available here: <https://www.dst.dk/pubfile/22257/appendix>.

the data, we also assume that firms only operate in one sector and assign the firm's most common sector as the only sector.

## **B Regression tables**

Table 2: Age coefficients from OLS regression

	(1)		(2)		(3)	
	emp_fte_log	(.)	turn_log	(.)	va_log	(.)
age_trunc_var=1	0.00	(.)	0.00	(.)	0.00	(.)
age_trunc_var=2	0.85***	(290.11)	0.67***	(211.58)	0.63***	(225.84)
age_trunc_var=3	0.90***	(289.04)	0.73***	(213.16)	0.71***	(228.51)
age_trunc_var=4	0.95***	(282.80)	0.78***	(210.49)	0.79***	(230.49)
age_trunc_var=5	0.99***	(276.83)	0.83***	(207.21)	0.85***	(229.54)
age_trunc_var=6	1.04***	(272.51)	0.88***	(206.57)	0.91***	(229.55)
age_trunc_var=7	1.09***	(263.99)	0.94***	(201.35)	0.96***	(222.44)
age_trunc_var=8	1.11***	(258.74)	0.96***	(197.88)	1.00***	(219.66)
age_trunc_var=9	1.14***	(253.57)	0.99***	(196.38)	1.03***	(216.76)
age_trunc_var=10	1.16***	(248.70)	1.01***	(192.53)	1.05***	(213.21)
age_trunc_var=11	1.19***	(244.99)	1.04***	(189.12)	1.09***	(211.56)
age_trunc_var=12	1.21***	(239.53)	1.05***	(185.91)	1.11***	(207.46)
age_trunc_var=13	1.22***	(232.85)	1.06***	(179.59)	1.12***	(200.96)
age_trunc_var=14	1.23***	(228.08)	1.07***	(175.17)	1.13***	(196.78)
age_trunc_var=15	1.25***	(221.09)	1.09***	(169.93)	1.15***	(191.16)
age_trunc_var=16	1.25***	(216.61)	1.09***	(166.53)	1.15***	(186.80)
age_trunc_var=17	1.26***	(210.95)	1.09***	(161.53)	1.16***	(181.95)
age_trunc_var=18	1.26***	(206.60)	1.10***	(157.92)	1.16***	(177.93)
age_trunc_var=19	1.27***	(200.73)	1.10***	(153.47)	1.16***	(173.30)
age_trunc_var=20	1.27***	(195.13)	1.10***	(147.66)	1.17***	(168.44)
age_trunc_var=21	1.28***	(191.03)	1.11***	(144.89)	1.18***	(164.13)
age_trunc_var=22	1.29***	(186.79)	1.11***	(140.72)	1.19***	(160.60)
age_trunc_var=23	1.30***	(180.40)	1.12***	(135.78)	1.20***	(155.11)
age_trunc_var=24	1.30***	(173.97)	1.11***	(130.15)	1.19***	(149.08)
age_trunc_var=25	1.30***	(166.86)	1.11***	(125.25)	1.20***	(143.46)
age_trunc_var=26	1.29***	(161.44)	1.11***	(121.39)	1.19***	(138.44)
age_trunc_var=27	1.30***	(155.37)	1.11***	(116.74)	1.20***	(133.89)
age_trunc_var=28	1.31***	(149.70)	1.12***	(112.68)	1.22***	(129.69)
age_trunc_var=29	1.31***	(147.32)	1.12***	(109.89)	1.21***	(126.31)
age_trunc_var=30	1.32***	(142.26)	1.14***	(106.75)	1.23***	(122.15)
age_trunc_var=31	1.35***	(136.77)	1.16***	(103.06)	1.26***	(118.58)
age_trunc_var=32	1.37***	(131.80)	1.18***	(98.97)	1.29***	(114.74)
age_trunc_var=33	1.39***	(125.82)	1.20***	(94.87)	1.31***	(110.24)
age_trunc_var=34	1.40***	(119.87)	1.20***	(89.56)	1.32***	(103.35)
age_trunc_var=35	1.43***	(113.48)	1.21***	(83.34)	1.35***	(98.44)
age_trunc_var=36	1.48***	(108.03)	1.26***	(79.79)	1.40***	(94.69)
age_trunc_var=37	1.51***	(102.89)	1.26***	(73.88)	1.43***	(89.60)
age_trunc_var=38	1.56***	(98.26)	1.30***	(70.21)	1.47***	(84.78)
age_trunc_var=39	1.59***	(92.75)	1.30***	(64.91)	1.51***	(81.21)
age_trunc_var=40	1.62***	(88.27)	1.32***	(60.72)	1.53***	(76.00)
age_trunc_var=41	1.65***	(83.02)	1.32***	(56.37)	1.56***	(71.80)
age_trunc_var=42	1.68***	(79.01)	1.35***	(54.28)	1.59***	(68.46)
age_trunc_var=43	1.70***	(76.43)	1.38***	(52.61)	1.61***	(65.38)
age_trunc_var=44	1.75***	(72.83)	1.39***	(49.03)	1.68***	(63.92)
age_trunc_var=45	1.80***	(68.87)	1.47***	(48.02)	1.74***	(61.10)
age_trunc_var=46	1.84***	(64.36)	1.50***	(43.97)	1.80***	(57.00)
age_trunc_var=47	1.89***	(59.71)	1.54***	(41.29)	1.86***	(53.20)
age_trunc_var=48	1.94***	(56.65)	1.59***	(38.94)	1.90***	(50.01)
age_trunc_var=49	2.02***	(50.00)	1.68***	(35.42)	1.99***	(45.19)
age_trunc_var=50	2.15***	(48.55)	1.71***	(32.06)	2.11***	(42.95)
age_trunc_var=51	2.08***	(44.46)	1.77***	(32.79)	2.08***	(41.48)
age_trunc_var=52	2.14***	(42.84)	1.83***	(30.73)	2.15***	(39.47)
age_trunc_var=53	2.15***	(38.80)	1.80***	(27.84)	2.17***	(36.22)
age_trunc_var=54	2.13***	(39.00)	1.75***	(27.07)	2.14***	(36.10)
age_trunc_var=55	2.23***	(38.40)	2.01***	(29.18)	2.18***	(34.07)
age_trunc_var=56	2.22***	(36.65)	2.13***	(29.57)	2.37***	(36.68)
age_trunc_var=57	2.38***	(34.84)	2.30***	(29.41)	2.42***	(33.08)
age_trunc_var=58	2.38***	(32.33)	2.41***	(29.42)	2.51***	(33.25)
age_trunc_var=59	2.46***	(31.43)	2.61***	(30.48)	2.74***	(35.25)
age_trunc_var=60	2.84***	(27.19)	2.93***	(25.27)	2.93***	(25.01)
age_trunc_var=61	2.86***	(25.58)	2.88***	(23.01)	2.99***	(24.97)
age_trunc_var=62	2.96***	(24.31)	2.99***	(21.72)	3.15***	(24.78)
age_trunc_var=63	2.89***	(21.92)	2.95***	(20.92)	3.09***	(23.18)
age_trunc_var=64	2.86***	(18.63)	2.94***	(18.29)	3.04***	(20.08)
age_trunc_var=65	3.07***	(18.37)	3.16***	(17.43)	3.23***	(19.25)
age_trunc_var=66	2.92***	(14.71)	3.05***	(14.60)	3.10***	(15.20)
D = 1	-1.24***	(-189.57)	-2.53***	(-318.75)	-1.34***	(-182.86)
Constant	0.56***	(178.87)	8.02***	(2589.65)	7.04***	(2277.64)
Observations	1839145		1880540		1795889	
Adjusted R <sup>2</sup>	0.238		0.342		0.235	
regression	REGHDFE		REGHDFE		REGHDFE	
clustering	firm		firm		firm	
time FE	yes		yes		yes	
sector FE	yes		yes		yes	

t statistics in parentheses

\* p &lt; 0.10, \*\* p &lt; 0.05, \*\*\* p &lt; 0.01

Note: Standard errors are clustered at firm level. Age of firm is equal to age\_trunc\_var-1.

Table 3: Age coefficients from FE regression

	(1)		(2)		(3)	
	emp_fte_log		turn_log		va_log	
	0.00	(.)	0.00	(.)	0.00	(.)
age_trunc_var=1						
age_trunc_var=2	0.53***	(240.12)	0.43***	(176.26)	0.39***	(172.60)
age_trunc_var=3	0.61***	(238.34)	0.49***	(179.49)	0.46***	(178.00)
age_trunc_var=4	0.65***	(234.19)	0.52***	(175.09)	0.50***	(179.54)
age_trunc_var=5	0.67***	(230.58)	0.53***	(169.09)	0.52***	(176.86)
age_trunc_var=6	0.69***	(228.02)	0.54***	(165.36)	0.54***	(173.05)
age_trunc_var=7	0.70***	(221.70)	0.55***	(159.04)	0.55***	(165.10)
age_trunc_var=8	0.72***	(215.96)	0.57***	(154.89)	0.57***	(161.12)
age_trunc_var=9	0.73***	(209.53)	0.58***	(151.95)	0.57***	(155.00)
age_trunc_var=10	0.74***	(205.07)	0.59***	(147.80)	0.58***	(150.64)
age_trunc_var=11	0.75***	(199.10)	0.60***	(143.03)	0.59***	(146.20)
age_trunc_var=12	0.76***	(193.86)	0.60***	(139.73)	0.60***	(141.92)
age_trunc_var=13	0.76***	(189.12)	0.60***	(134.04)	0.59***	(135.68)
age_trunc_var=14	0.77***	(183.28)	0.60***	(129.48)	0.59***	(130.62)
age_trunc_var=15	0.77***	(175.36)	0.60***	(124.10)	0.59***	(124.35)
age_trunc_var=16	0.77***	(170.60)	0.60***	(119.90)	0.59***	(119.42)
age_trunc_var=17	0.77***	(164.01)	0.60***	(114.37)	0.58***	(114.63)
age_trunc_var=18	0.77***	(158.39)	0.60***	(110.86)	0.58***	(110.69)
age_trunc_var=19	0.77***	(152.79)	0.61***	(107.85)	0.59***	(106.63)
age_trunc_var=20	0.77***	(147.97)	0.60***	(102.97)	0.58***	(102.64)
age_trunc_var=21	0.77***	(143.04)	0.60***	(98.99)	0.57***	(97.41)
age_trunc_var=22	0.76***	(138.26)	0.59***	(94.54)	0.57***	(93.39)
age_trunc_var=23	0.76***	(132.61)	0.58***	(89.99)	0.56***	(88.18)
age_trunc_var=24	0.75***	(128.28)	0.58***	(85.74)	0.55***	(83.61)
age_trunc_var=25	0.75***	(122.41)	0.57***	(81.69)	0.54***	(79.19)
age_trunc_var=26	0.74***	(117.82)	0.57***	(78.81)	0.53***	(75.32)
age_trunc_var=27	0.73***	(112.52)	0.56***	(73.59)	0.52***	(72.18)
age_trunc_var=28	0.73***	(108.65)	0.55***	(71.30)	0.52***	(69.09)
age_trunc_var=29	0.71***	(103.28)	0.54***	(66.61)	0.50***	(64.54)
age_trunc_var=30	0.71***	(100.19)	0.53***	(64.29)	0.49***	(61.88)
age_trunc_var=31	0.70***	(95.48)	0.53***	(61.54)	0.49***	(59.14)
age_trunc_var=32	0.69***	(91.60)	0.52***	(57.95)	0.48***	(56.80)
age_trunc_var=33	0.68***	(86.46)	0.51***	(54.70)	0.47***	(52.93)
age_trunc_var=34	0.66***	(81.82)	0.48***	(48.95)	0.44***	(47.98)
age_trunc_var=35	0.64***	(75.44)	0.46***	(44.58)	0.43***	(44.29)
age_trunc_var=36	0.63***	(70.54)	0.45***	(41.23)	0.41***	(40.35)
age_trunc_var=37	0.62***	(67.10)	0.41***	(36.16)	0.39***	(37.03)
age_trunc_var=38	0.60***	(60.70)	0.39***	(31.96)	0.37***	(32.92)
age_trunc_var=39	0.59***	(56.72)	0.36***	(27.95)	0.37***	(31.18)
age_trunc_var=40	0.57***	(53.29)	0.36***	(26.58)	0.35***	(28.08)
age_trunc_var=41	0.55***	(47.63)	0.34***	(23.01)	0.33***	(25.25)
age_trunc_var=42	0.54***	(44.35)	0.32***	(20.72)	0.32***	(22.71)
age_trunc_var=43	0.52***	(40.19)	0.32***	(19.59)	0.31***	(20.92)
age_trunc_var=44	0.51***	(36.74)	0.30***	(16.95)	0.31***	(20.11)
age_trunc_var=45	0.51***	(35.16)	0.31***	(17.22)	0.31***	(18.92)
age_trunc_var=46	0.51***	(33.64)	0.30***	(14.80)	0.31***	(17.55)
age_trunc_var=47	0.49***	(29.29)	0.31***	(15.35)	0.31***	(16.12)
age_trunc_var=48	0.48***	(27.26)	0.25***	(11.03)	0.27***	(13.14)
age_trunc_var=49	0.47***	(22.85)	0.25***	(9.48)	0.26***	(11.06)
age_trunc_var=50	0.46***	(21.92)	0.21***	(7.81)	0.25***	(9.91)
age_trunc_var=51	0.46***	(19.61)	0.24***	(8.38)	0.25***	(9.79)
age_trunc_var=52	0.44***	(17.66)	0.23***	(7.61)	0.23***	(8.28)
age_trunc_var=53	0.44***	(16.99)	0.25***	(7.55)	0.24***	(7.85)
age_trunc_var=54	0.42***	(14.34)	0.18***	(5.03)	0.18***	(5.77)
age_trunc_var=55	0.42***	(14.53)	0.20***	(5.73)	0.15***	(4.53)
age_trunc_var=56	0.40***	(12.74)	0.30***	(7.92)	0.29***	(8.44)
age_trunc_var=57	0.40***	(11.28)	0.28***	(7.14)	0.18***	(4.91)
age_trunc_var=58	0.38***	(10.19)	0.33***	(8.24)	0.23***	(5.99)
age_trunc_var=59	0.38***	(9.48)	0.43***	(8.97)	0.38***	(8.73)
age_trunc_var=60	0.27***	(4.79)	0.23***	(3.96)	0.13**	(2.16)
age_trunc_var=61	0.28***	(4.71)	0.19***	(2.74)	0.18***	(2.67)
age_trunc_var=62	0.25***	(4.24)	0.22***	(3.14)	0.21***	(3.31)
age_trunc_var=63	0.21***	(3.17)	0.16***	(2.20)	0.17***	(2.39)
age_trunc_var=64	0.19***	(2.59)	0.19**	(2.49)	0.14*	(1.71)
age_trunc_var=65	0.27***	(4.11)	0.19**	(2.38)	0.19**	(2.36)
age_trunc_var=66	0.19***	(2.58)	0.16*	(1.77)	0.13	(1.40)
Δ GDP	0.40***	(24.03)	0.76***	(36.65)	1.00***	(51.56)
D = 1	-0.97***	(-162.58)	-2.04***	(-252.33)	-0.97***	(-151.59)
Constant	0.96***	(322.24)	8.39***	(2736.36)	7.48***	(2379.48)
Observations	1839145		1880540		1795889	
Adjusted R <sup>2</sup>	0.172		0.281		0.108	
regression	xtreg		xtreg		xtreg	
clustering	firm		firm		firm	
time FE	no		no		no	
sector FE	no		no		no	

t statistics in parentheses

\* p &lt; 0.10, \*\* p &lt; 0.05, \*\*\* p &lt; 0.01

Note: Standard errors are clustered at firm level. Age of firm is equal to age\_trunc\_var-1. Δ GDP is the percentage aggregated GDP growth rate.

## C Robustness and results with alternative specifications

### C.1 Results for growth rates

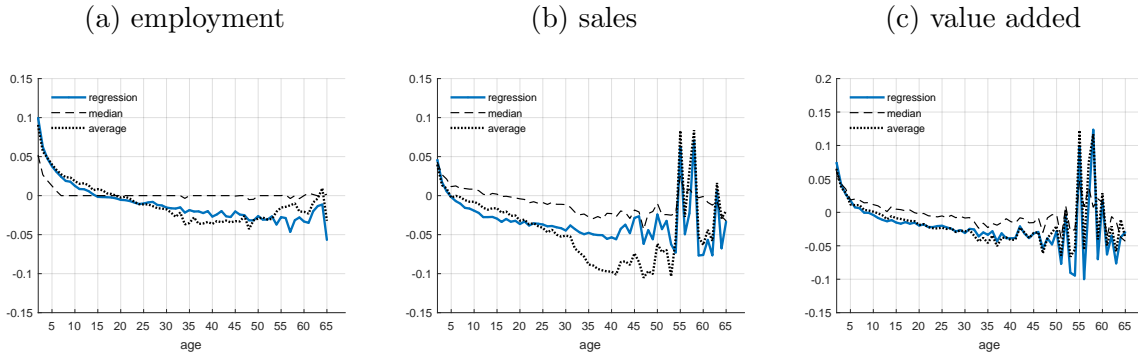
Here we show the results for growth rates instead of log levels

When discussing the effects of age on variable  $z$ , we present the results for log-levels, and for growth rates, which are computed as normalized growth rates, as suggested by [Haltiwanger et al. \(2013\)](#):

$$\hat{g}_{z_{i,t}} \equiv \frac{z_{i,t} - z_{i,t-1}}{\frac{1}{2}(z_{i,t} + z_{i,t-1})}, \quad (6)$$

where  $i$  is the firm index and  $t$  is the time index. We choose this normalization of growth rate because of the following advantages. First, it is always bounded between -2 and 2. If we interpret entry as moving from zero to particular positive value and vice versa if we imagine exit as moving from any positive number to zero, we see that entering firms can get assign growth rate of 2 and exiting firms get -2. Second, this measure of growth rate is symmetric, in the sense that a firm growing from size  $a$  to  $b$  in one period and then from  $b$  back to  $a$  has the same absolute value, but just with different sign. Finally, for a small deviations from zero, it approximates log difference.

Figure 9: Age profiles - growth rates

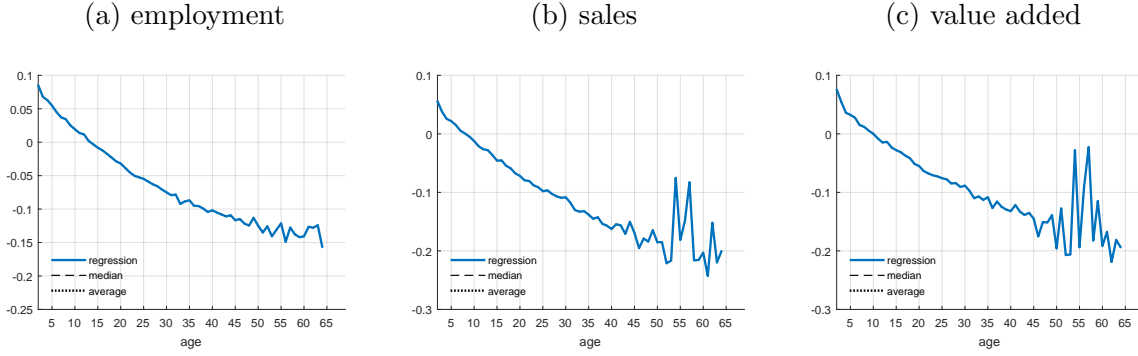


*Note:* This figure shows the unconditional mean and median age pattern (black) as well as the predicted age pattern based on regression (1).

Figure 9 displays the average growth rate of the firms across the age distribution. We see that for growth rates, in contrast to levels, the average growth rate falls with age in

the cross section.

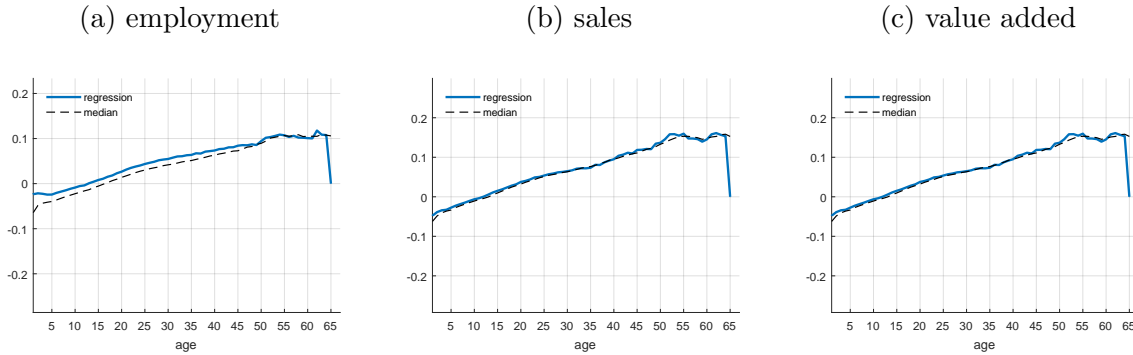
Figure 10: Age profiles with firm fixed effects, growth rates



*Note:* The figure displays the predicted age pattern from regression (2). Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of Danish kroner.

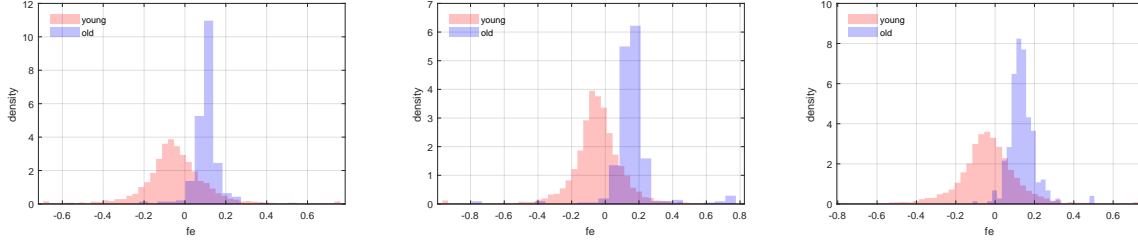
But are there also firm-inherent differences in the rate of firm growth that are systematically related to the likelihood of survival? The analyses in this section show that this is indeed the case: Firms that survive longer grow on average at a higher rate across their life-cycle. This documented by the observation that the distribution of firm fixed effects in growth rates shifts to the right for older ages (Figure 12) and that the average firm fixed effect in growth rates increases with age as shown in Figure 11.

Figure 11: Age profiles of firm fixed effects, growth rates



*Note:* The figure displays the average fixed effects estimated from regression (2) where the outcome variables are employment (column 1) or sales (column 2), either in log levels (row 1) or growth rates as defined in equation (6) (row 2).

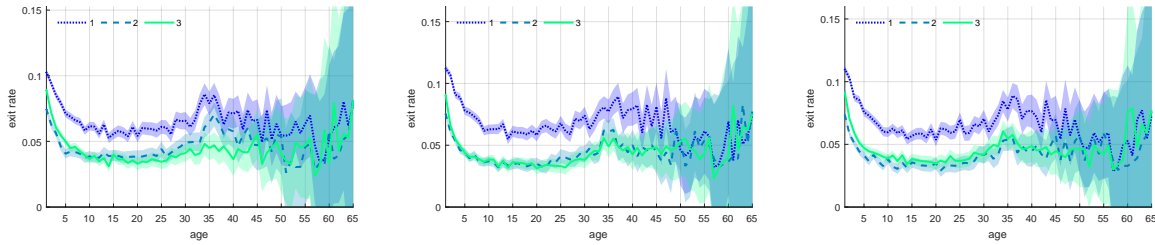
Figure 12: Histograms of firm FE, growth rates  
(a) employment (b) sales (c) value added



*Note:* The figure displays the density functions of the firm fixed effects  $\hat{\mu}_i$  estimated from equation (2) for two age groups: the ‘young’ firms of age 3–5 years (red) and the ‘old’ firms of 50 years and older (blue). Outcome variables are employment (first column) and sales (second column), logs (first row) or growth rates (second row). Firm fixed effects are winsorized at 1 and 99%.

Figure 13: Exit rates by position in the firm distribution, growth rates

(a) employment (b) sales (c) value added



*Note:* The figure displays the predicted exit probabilities by age from equation (3) for different size terciles (first tercile: dotted blue line, second tercile: dashed dark green line, third tercile: solid light green) for employment (panel (a)), sales (b) and value added (c). The shaded areas correspond to 95% confidence intervals.

Figure 14: APC 0 - only nonlinear components, growth rates

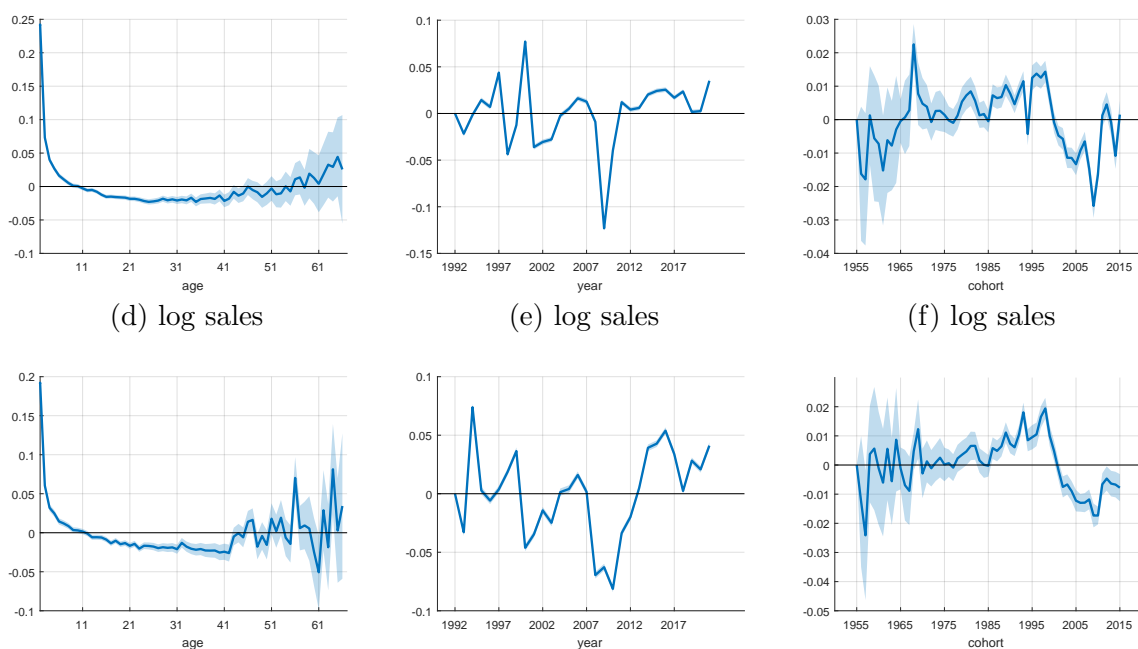
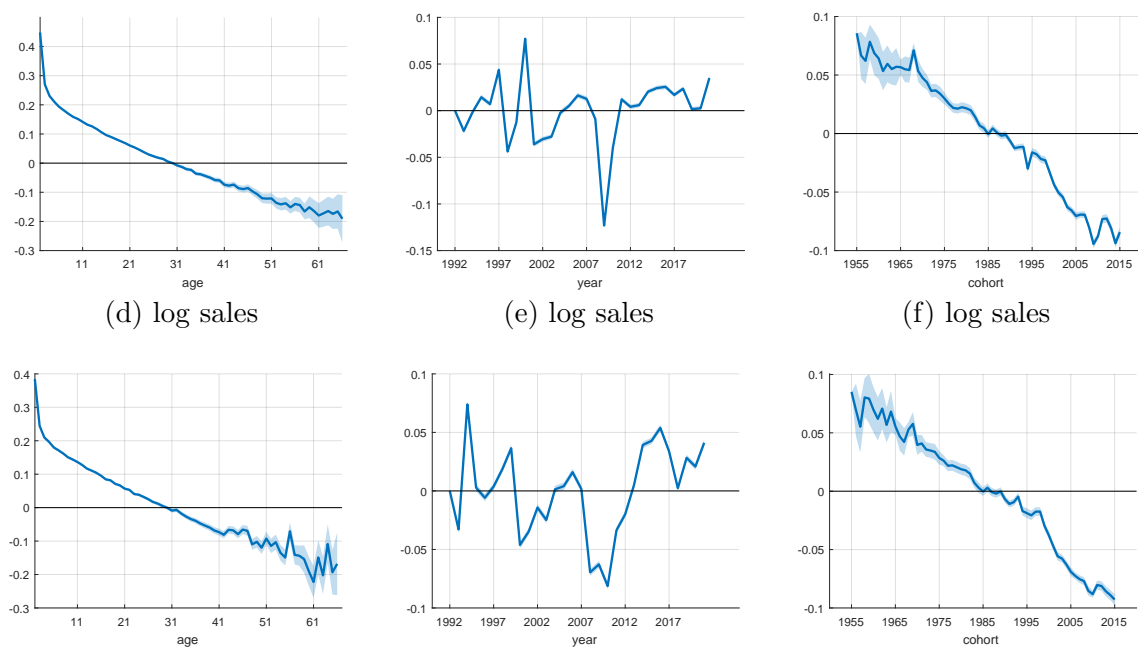


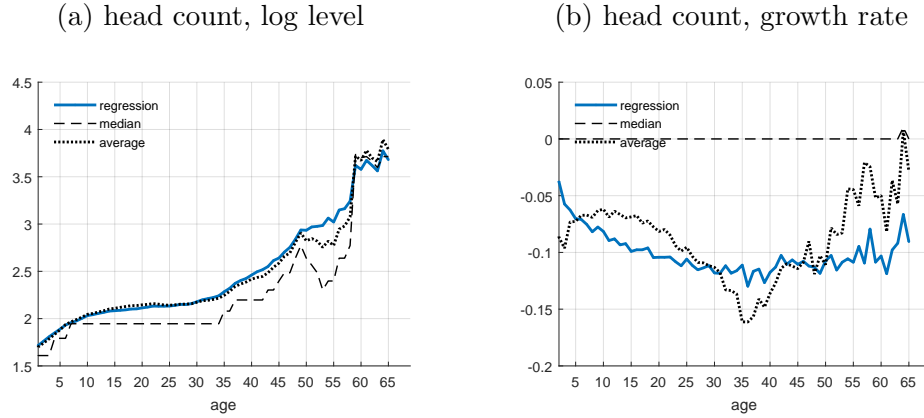
Figure 15: APC II - no trend in period, growth rates





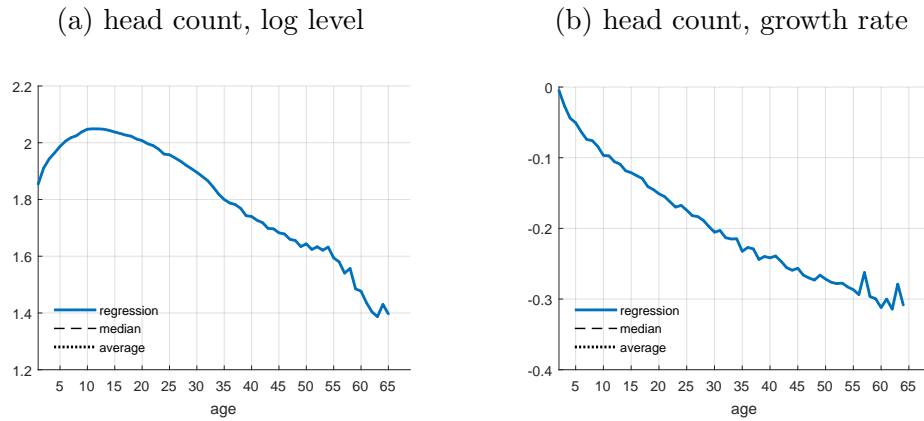
## C.2 Alternative definitions of size

Figure 16: Age profiles - all firms



*Note:* This figure shows the unconditional mean and median (black) as well as the predicted age pattern based on regression (1) (blue).

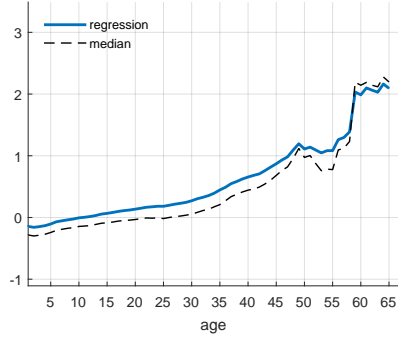
Figure 17: Age profiles with firm fixed effects, log levels



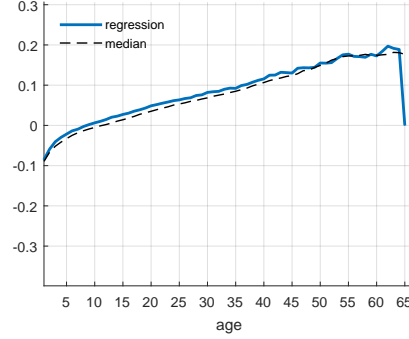
*Note:* The figure displays the predicted age pattern from regression (2). Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of Danish kroner.

Figure 18: Age profiles with firm fixed effects, log levels

(a) head count, log level



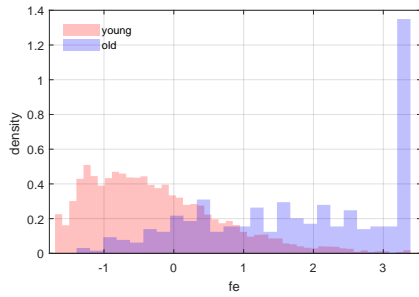
(b) head count, growth rate



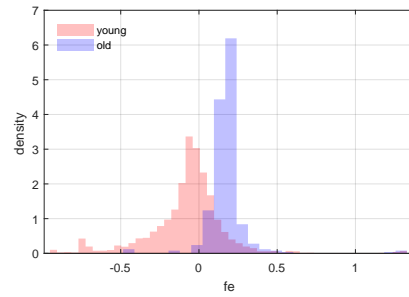
*Note:* The figure displays the average fixed effects estimated from regression (2) where the outcome variables are employment (column 1), value added (column 2) and turnover (column 3), either in levels (row 1), logs (row 2) or growth rates as defined in equation (6) (row 3). Shaded areas refer to 95% confidence intervals. Value added and turnover are measured in thousands of 2020 Danish kroner.

Figure 19: Histograms of firm FE

(a) head count, log level



(b) head count, growth rate

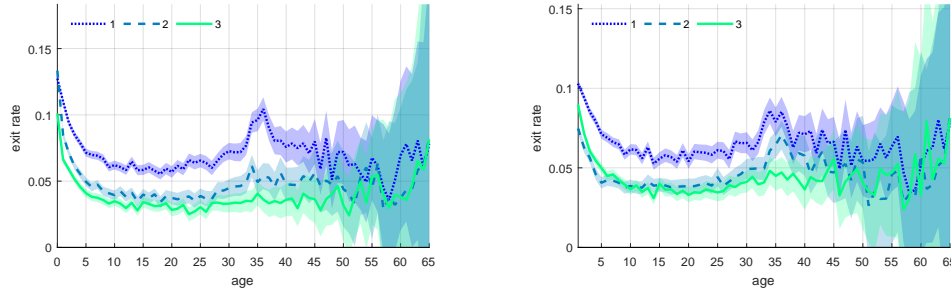


*Note:* The figure displays the density functions of the firm fixed effects  $\hat{\mu}_i$  estimated from equation (2) for two age groups: the ‘young’ firms of age 3–5 years (red) and the ‘old’ firms of 50 years and older (blue). Outcome variables are headcount (first column) and value added (second column), logs (first row) or growth rates (second row). Firm fixed effects are winsorized at 1 and 99%, noticeably, the upper threshold is much more binding for the older firms when the regression (2) is estimated in log-levels rather than growth rates.

Figure 20: Exit rates by position in the firm distribution

(a) head count, log level

(b) head count, growth rate



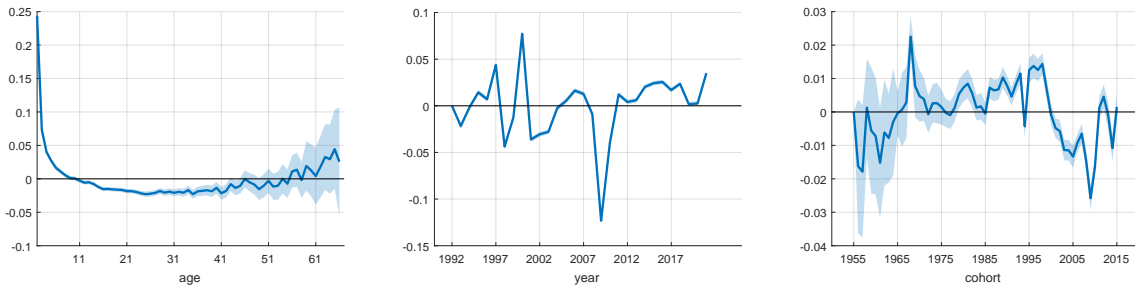
*Note:* The figure displays the predicted exit probabilities by age from equation (3) for different size terciles (first tercile: dotted blue line, second tercile: dashed dark green line, third tercile: solid light green) for employment (panel (a)), sales (b) and value added (c). The shaded areas correspond to 95% confidence intervals.

Figure 21: APC 0 - only nonlinear components

(a) log headcount

(b) log headcount

(c) log headcount



### C.3 Alternative groups definition for FE histogram

Figure 22: Firm FE histogram



*Note:* The figure displays the density functions of the firm fixed effects  $\hat{\mu}_i$  estimated from equation (2) for two age groups: the ‘young’ firms of age 3–5 years (red) and the ‘old’ firms of 50 years and older (blue). Outcome variables are employment (first column) and sales (second column), logs (first row) or growth rates (second row). Firm fixed effects are winsorized at 0.5 and 99.5%.

### C.4 Firm exit when using firm FE

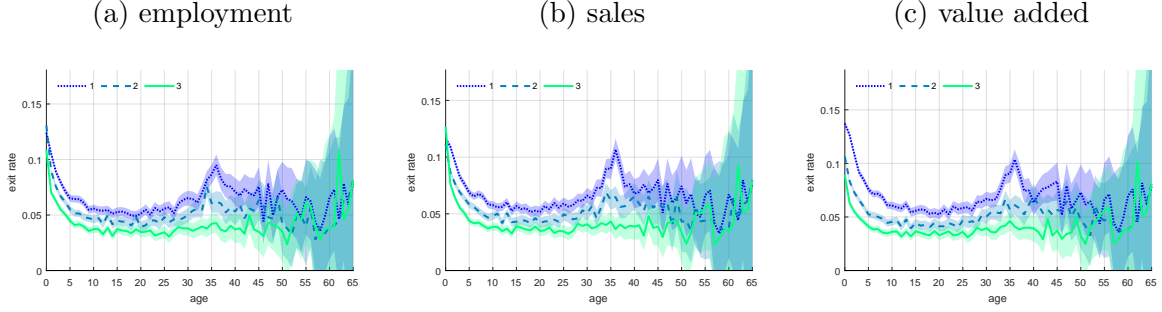
As an alternative to using the current size as a (possible) determinant for firm exit rate (as we did in regression 3), here we show that the results also hold when we use the firm fixed effect instead. Specifically, we use the following regression:

$$D_{it} = \mu + \sum_{j=1}^3 \sum_{a=0}^{N_a} \alpha_a 1_{(A_{it}=a)} \times Q_{it}^{FEY}(j) + \sum_{p=0}^{N_p} \pi_p 1_{(t=p)} + \varepsilon_{it}, \quad (7)$$

where  $Q_{it}^{FEY}(j)$  is an indicator function based on tercile of distribution of firm fixed effect  $\mu_i$  from regression (2). The tercile is computed within sector, so we do not include sector

in this regression. Also, we do not include time fixed

Figure 23: Exit rates by position in the firm FE distribution



*Note:* The figure displays the predicted exit probabilities by age from equation (3) for different terciles of estimated firm FE  $\hat{\mu}$  from Equation (2) for employment (panel (a)), sales (b) and value added (c).

## C.5 More background and results for the APC

Using the notation of [Fosse and Winship \(2019, page 1979 and onwards\)](#), we can think of decomposing outcome  $Y$  into age, period and cohort components:

$$Y_{ijk} = \mu + \alpha_i + \pi_j + \gamma_k + \varepsilon_{ijk}.$$

One can separate the apc terms into slopes and the deviations from the slopes:  $\alpha_i = \alpha(a_i - a^*) + \tilde{\alpha}_i$ ,  $\pi_j = \pi(p_j - p^*) + \tilde{\pi}_j$  and  $\gamma_k = \gamma(k - k^*) + \tilde{\gamma}_k$ , so that anytime a firm ages by one year, the effect is  $\alpha$ , additive to the nonlinear effect of a particular age  $\tilde{\alpha}_i$  and similarly for the other components. This yields

$$Y_{ijk} = \mu + \alpha(a_i - a^*) + \pi(p_i - p^*) + \gamma(k_i - k^*) + \tilde{\alpha}_a + \tilde{\pi}_p + \tilde{\gamma}_k + \varepsilon_{ijk}$$

The problem with separate identification of age, period and cohort effects comes from the fact that  $period_j = cohort_k + age_i$  (or  $p_i = k_i + a_i$ ) by definition, so only two parameters out of all three  $(\alpha, \pi, \gamma)$  can be identified. It manifests itself as the regressors matrix not having full rank. Moreover, if  $\{\alpha, \pi, \gamma\}$  is a solution to the system of equation described above, then for any  $\nu$ ,  $\{\alpha + \nu, \pi - \nu, \gamma + \nu\}$  also solves it. However, certain linear combinations of  $(\alpha, \pi, \gamma)$  can be identified, obtained via regression and prove useful. Putting the nonlinear

effects aside, one can write the previous equation as

$$\begin{aligned}
Y_{ijk} &= \mu + \alpha age_i + \pi period_j + \gamma cohort_k + \varepsilon_{ijk} \\
Y_{ijk} &= \mu + \alpha age_i + \pi period_j + \gamma(period_j - age_i) + \varepsilon_{ijk} \\
Y_{ijk} &= \mu + (\alpha - \gamma)age_i + (\gamma + \pi)period_j + \varepsilon_{ijk} \\
&= \mu + \omega_1 age_i + \omega_2 period_j + \varepsilon_{ijk}
\end{aligned} \tag{8}$$

Imposing a constraint (either in form of an inequality or in a form of an interval) on one of  $\{\alpha, \pi, \gamma\}$  (or a combination of couple) also puts constraints on the other (partial identification) because

$$\begin{aligned}
\alpha - \gamma &= \omega_1 \\
\gamma + \pi &= \omega_2.
\end{aligned} \tag{9}$$

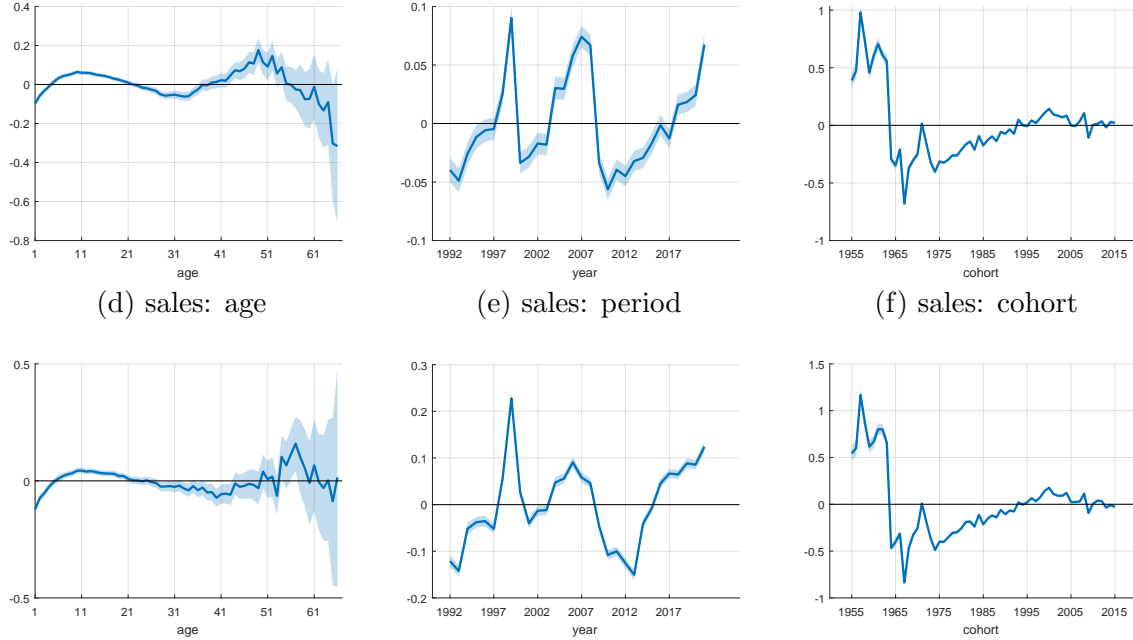
While the three APC linear trend coefficients cannot be separately identified, if we believe the value of a particular one (or a range for it), it implies constraints on the other. For example, if we believe that the true value  $\alpha$  is  $\hat{\alpha}$ , it then also has to be the case that that  $\gamma = \hat{\alpha} - \omega_1$  and so  $\pi = \omega_2 - \gamma = \omega_2 + \omega_1 - \hat{\alpha}$ . Alternatively, and perhaps more realistically, one can impose inequalities. Just as in the case of equalities, adding additional constraint in form of inequality on one coefficient implies inequality constraint on the other coefficients via the system of Equations (9).

Using this insight, [Fosse and Winship \(2019\)](#) suggest the following approach in practice. First, estimate  $\{\theta_1, \theta_2\}$  from regression along the lines of Equation (8) that also include the non-linear terms  $\tilde{\alpha}_i$ ,  $\tilde{\pi}_j$  and  $\tilde{\gamma}_k$ . With these coefficients estimated, one can impose additional constraints from intuition or outside knowledge of the problem and use Equations (9) to get values (or sets of values) for the other coefficients.

It is also practical to re-center the age, cohort and period variables such that the new variable is equal to 0 at the mid point (as in if years go from 1992-2022, the new period variable will go from -15 to 15). That way, one can visualize the effect of adding the trend to the nonlinear components as rotating the pattern of the nonlinear components.

Both employment and sales share qualitatively very similar patterns for age, cohort and period nonlinear effects. Starting with the cohort effects, *before* adding any possible cohort trend, one can see that the firms that started in the 50's were much better than the firms that started in the late 60's. The non-linear period effects show strong drop around the

Figure 24: APC estimation results: nonlinear components  
(a) employment: age (b) employment: period (c) employment: cohort



*Note:* This Figure show the results of estimating Equation (4), namely  $\alpha_a$  (panel (a) for log level of employment and panel (d) for log level of sales)  $\pi_p$  (panel (b) for log level of employment and panel (e) for log level of sales),  $\gamma_c$  (panel (c) for log level of employment and panel (f) for log level of sales). The shaded areas correspond to 95% confidence intervals.

great financial crisis and a peak in 1999. The age nonlinear-effects show a gradual increase for the first 10 years or so, followed by a region with negative slope until 30-40 years at which point the slope becomes positive again. For employment the pattern reverses again around 50.

Figure 25: APC I - no trend in cohorts

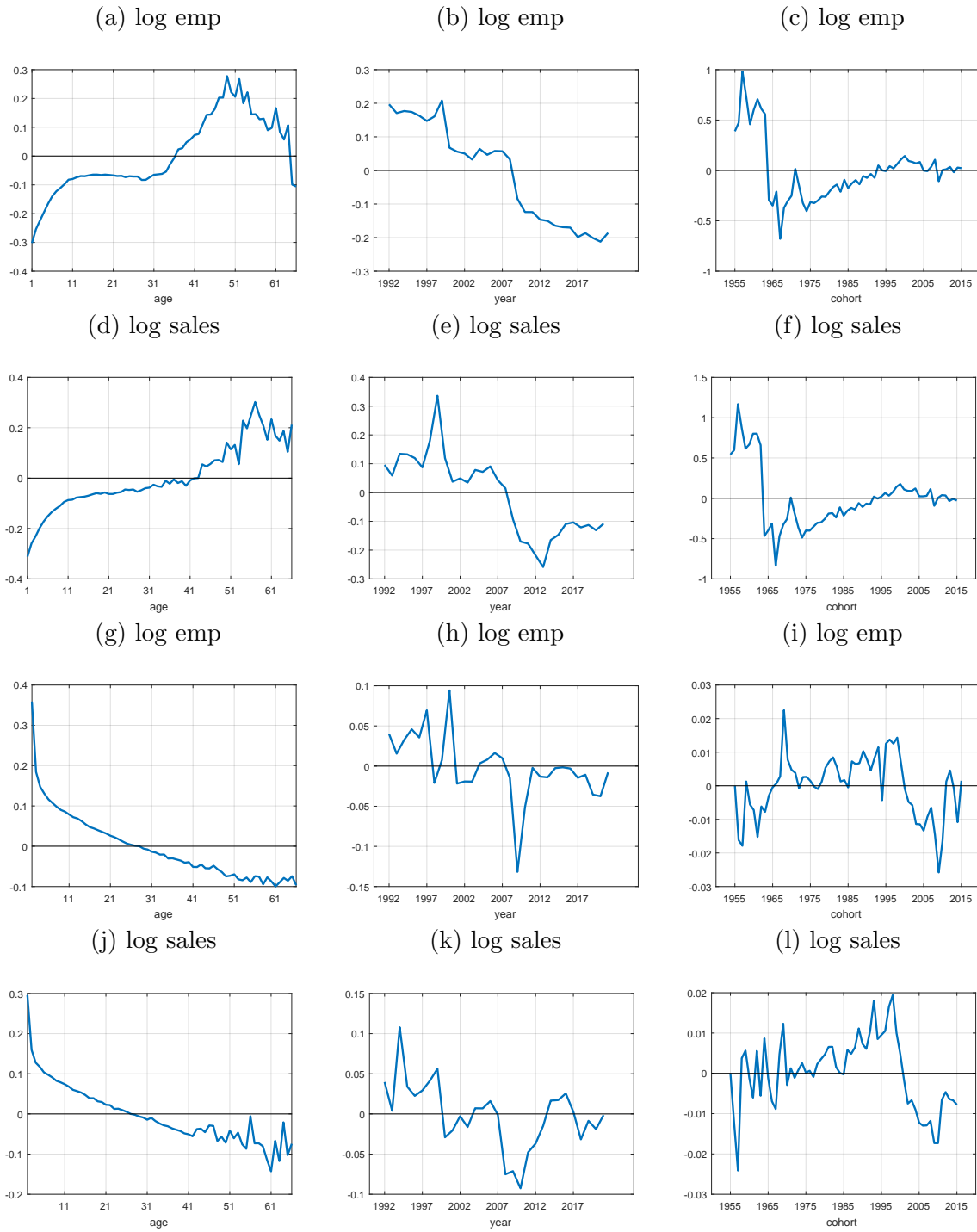




Figure 26: APC III - no trend in age

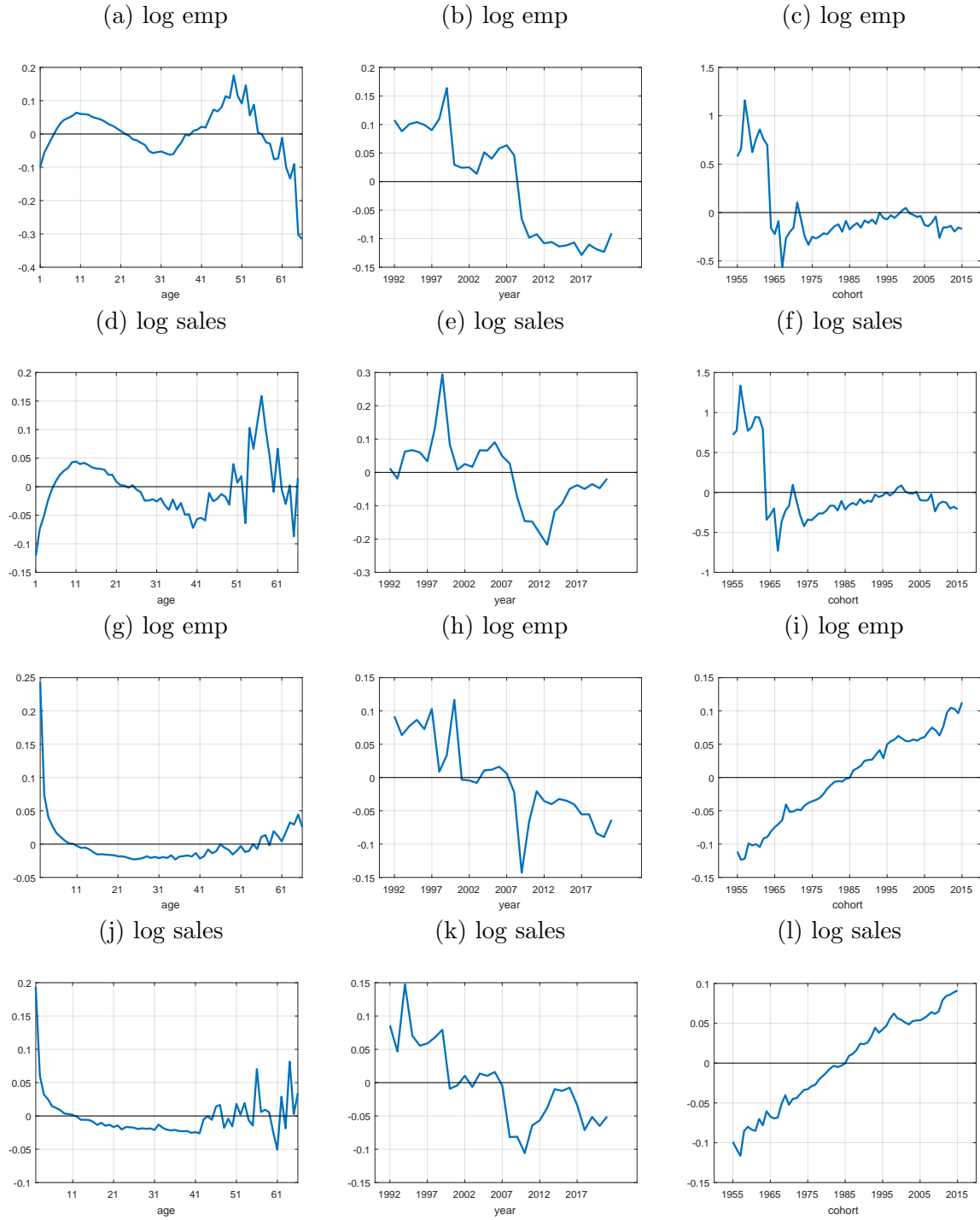


Figure 27: APC IV - flat profile in age after 50

